DEVELOPING ACCEPTANCE-SAMPLING METHODS FOR QUALITY CONSTRUCTION

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ABSTRACT: An acceptance-sampling method plays an important role in designing quality-assurance (QA) specifications. It applies statistics to specify the requirements of (1) how many measurements are needed; and (2) how to make an acceptance or rejection decision based on measured data. The key function of the acceptance-sampling method is to guide the decision between accepting or rejecting the quality of the products. The purpose of this paper is to present four acceptance-sampling methods, including (1) variable–single-sampling method; (2) quality-index–sampling method; (3) attribute–double-sampling method; and (4) attribute–proportion-sampling method. In this paper, the theories will be derived and the applications of the four developed quality-acceptance–sampling methods will be described. The advantages and disadvantages of using different sampling methods are compared. The application of the developed quality-acceptance-sampling methods is illustrated using bridge-painting examples.

INTRODUCTION

The state highway agencies have increasingly demanded quality construction in recent years. Quality-assurance (QA) specification systems have been advocated and developed by many state highway agencies throughout the nation (Weseman and Erickson 1988; Tewari 1986; Weed 1989; Willenbrock and Kopac 1977; Gendell and Masuda 1988; Gentry and Yrjanson 1987). Researchers have shown that billions of dollars can be saved each year by implementing QA specifications ("Asphalt" 1990; Bower 1991; Harp 1991). As a result, Purdue University and the Indiana Department of Transportation are cooperating with the Federal Highway Administration to develop a new QA specification for steel-bridge painting. The purpose of this paper is to present four acceptance-sampling methods proposed in the developed QA specification (Chang and Hsie 1992a).

An acceptance-sampling method (ASM) plays an important role in designing QA specifications (Burati and Hughes 1990; Lynn 1989; O'Brien 1989). The ASM applies statistics to specify the requirements of (1) how many measurements are needed; and (2) how to make an acceptance or rejection decision based on measured data. The key function of the ASM is to guide the decision between accepting or rejecting the quality of the products.

Several acceptance-sampling methods have been developed (Wadsworth et al. 1986). To distinguish acceptance-sampling methods, different categorizations are used. Depending on the number of samples taken, the ASM can be classified as single or double sampling. In the ways that the sampling data are utilized, ASM can be classified as an attribute or variable sampling. In this paper, four acceptance-sampling methods are presented, including (1) variable–single-sampling (VSS) method; (2) quality-index–sampling (QIS) method; (3) attribute–double-sampling (ADS) method; and (4) attribute–proportion-sampling (APS) method.

To facilitate the reader's understanding, some statistical terms will be explained first. Then, the theories will be derived, and the applications of the four developed quality-acceptance-sampling methods will be described. The advantages and disadvantages of using different sampling methods are compared. The application of the developed quality-acceptance-sampling methods is illustrated using bridge-painting examples.

TERMINOLOGY

To better describe the four acceptance methods in the following sections, the following terms need to be defined: lot; sample size; single/double sampling; attribute/variable sampling; operating characteristic (OC) curve, producer's/owner's risk; and quality index.

Lot and Sample Size

A lot is the basic unit of acceptance plans. Acceptance or rejection decisions are made within the lot. On many occasions, to reduce the costs of making a wrong decision from the acceptance plan, the whole finished product is divided into several small groups that are designated as lots. For instance, one member, several members, or all the members of the steel-bridge structure can be treated as a lot.

The number of the measurements is called the sample size and designated as n. Within a lot, one or several measurements are taken. All these n measurements are grouped and considered one sample.

Single and Double Sampling

In single-sampling plan, the decision to accept or reject a lot is made based on one sample. In double sampling, however, after the first sample is inspected, the lot may be accepted, rejected, or require a second sample to make the acceptance decision.

A double-sampling plan usually uses a smaller sample size. If the result from the first sample shows that the product obviously conforms to the specified requirements, the lot is accepted. However, if the first sampling shows that the product is obviously not conforming, then the lot is rejected. If the result from the first sample is between these two situations, a second sampling is required to obtain more information on the lot to make a final acceptance decision. The advantage of double sampling compared with single sampling is that it generally requires smaller sample sizes on the average to obtain the same efficiency as in a single-sampling plan (Wadsworth et al. 1986).

Attribute and Variable Sampling

Sampling plans can be categorized as attribute sampling and variable sampling. A distinguishing characteristic of var-

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iable sampling is the requirement of calculating the standard deviations (SD), which are used to depict the variations of the finished product. If it needs to calculate SD in the sampling plan, the plan is a variable-sampling plan; otherwise, it is an attribute-sampling plan.

An attribute-sampling plan usually produces one of only two possible results—the individual measurement in the sample either conforms or does not conform with the specified attribute. The following example is used to illustrate attribute versus variable sampling. The agency specifies that the measurements of primer dry film thickness (DFT) should be no less than 2.5 mils (6.35×10^{-2} mm; 1 mil = 2.54×10^{-2} mm). Assume that the DFT measurements from lot 1 are 1.5 mils (3.81×10^{-2} mm) and 2.6 mils (6.60×10^{-2} mm), and from lot 2 are 2.4 mils (6.10×10^{-2} mm) and 3.5 mils (8.89×10^{-2} mm). Using the attribute-sampling method, the results for these two lots are the same—one out of the two measurements is nonconforming; both would be considered 50% out of the limits. Intuitively, it is obvious that lot 2 with readings of 2.4 mils (6.10×10^{-2} mm) and 3.5 mils (8.89×10^{-2} mm) has a higher potential for satisfying the required lower limit of 2.5 mils (6.35×10^{-2} mm). However, with an attribute-sampling plan, the two sets of readings for lot 1 and lot 2 are the same; both are 50% out of limits. To gain better accuracy, a variable sampling can be used.

A variable-sampling plan makes better use of relevant information provided by inspected data. Instead of just determining whether an individual sample is within the specified limits, variable sampling utilizes the available data to estimate and represent the underlying population. After the overall population is estimated, a more accurate estimated percentage of nonconformance can be acquired. This statistical sampling method allows one to obtain the same level of discrimination power with smaller sample size (Wadsworth et al. 1986).

Operating Characteristic (OC) Curve

The OC curve is the curve that shows the probability of lot acceptance based on the various quality levels. Fig. 1 is an example of an OC curve. It is a very significant measurement: "One of the most useful considerations of a sampling plan is its operating characteristic (OC) function. Whenever a statistical sampling is derived, its description is not complete until its OC function or OC curve has been described" (Wadsworth et al. 1986). It shows that when the underlying quality level of the lot is 10% defective (or 90% conforming) with the specified requirements, by applying a specific acceptance plan, it can be estimated that this lot will be accepted with the chance of 95% (or rejected with the probability of 5%). When the underlying defective quality level of the lot goes up to 40%, the chance of accepting the lot decrease to 5%. When the acceptance plan changes, its OC curve will also change. Therefore, by setting a proper acceptance-sampling plan with different quality levels, the probability of accepting or rejecting the lot can be controlled and predicted.

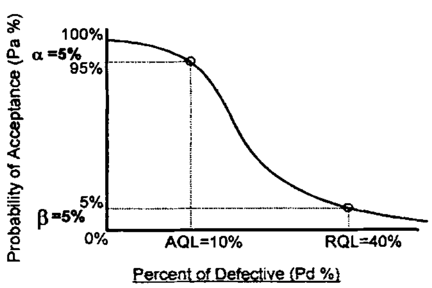

FIG. 1. Typical Operating-Characteristic (OC) Curve

Producer's/Owner's Risk

Contractor-supplied products that owners are willing to accept are designated acceptable quality level (AQL). However, because of the variation and uncertainty of sampling, it is not 100% guaranteed that all the samples taken from these AQL products will lead the owners to accept the products. The probability of rejecting AQL products is called the producer's risk, designated as α risk.

Products that owners are very sure to reject are designated as rejectable quality level (RQL). Likewise, it is not 100% guaranteed that these RQL products will be rejected based on samples taken. The probability that a RQL product will be accepted is called the owner's risk, designated as β risk. Fig. 1 shows the relation of AQL-α and RQL-β.

Statistics theory says that a more accurate estimate of quality can be obtained with larger sample sizes, thus reducing the risk of making a wrong decision. However, larger sample sizes will increase the cost of implementing an acceptance plan. How can people strike a balance between accuracy and cost? The solution is to analyze and control the risk of making a wrong acceptance-rejection decision. Therefore, many acceptance plans are developed based on controlling the α-AQL and β-RQL risks. In other words, these acceptance plans are designed to obtain the desired OC curves.

Quality Index

Quality index is a parameter used to estimate the percentage of defect (PD). In the construction industry, the measurement of quality can be costly. Therefore the sample size for each lot is likely to be small. Because the normal distribution is only appropriate for sample sizes of n greater than 30, to estimate the percentage out of the lower limit (or upper limit), the "non-central t" distribution should be utilized. Thus, instead of Z value (for normal distribution), the Q value, which is called the quality index, is used to obtain the PD. Tables for Q have been developed to assist in the calculation of PD (Wadsworth et al. 1986; Burati and Hughes 1990). The following example illustrates its use. Assume that the DFT lower limit (L) is 2.5 mils (6.35×10^{-2} mm); the sample size n is 10; the estimated mean value of the film thickness ($\bar{X}$) is 3.4 mils (8.64×10^{-2} mm); and the estimated standard deviation (SD) is 0.8 mils (2.03×10^{-2} mm). The quality index can be represented by the following equation:

$$Q = \frac{\bar{X} - L}{s} \approx \frac{\bar{X} - L}{SD} = \frac{3.4 - 2.5}{0.8} = 1.13 \quad (1)$$

By checking the Q value of 1.13 at $n = 10$ in the quality-index table, the estimated PD is 12.80%.

FOUR ACCEPTANCE-SAMPLING METHODS

Acceptance plans in quality-assurance programs vary because different statistical theories and combined quality parameters are applied to derive the decision mechanism. Four different methods are discussed in the following sections: (1) variable–single-sampling (VSS) method; (2) quality-index–sampling (QIS) method; (3) attribute–double-sampling (ADS) method; and (4) attribute–proportion-sampling (APS) method.

Variable–Single-Sampling (VSS) Method

Variable single sampling is derived by controlling two points that correspond to AQL and RQL on the OC curve. This theory is illustrated with the following example in the area of painting construction. Assume that there are two lots of finished painting products that need to be inspected. The agency specifies that the lower limit (L) for the primer film thickness be 2.5 mils (6.35×10^{-2} mm). A film thickness less than (L) 2.5 mils would be rated as defective or nonconforming. Assume that two lots have very different quality levels: lot 1 has an AQL of 10% defective; lot 2 has a RQL of 30% defective. The agency would like to develop an acceptance-sampling plan so that the designed acceptance plan would make the following decisions: (1) it would accept lot 1 with the probability of 95% (5% probability of rejection); and (2) it would accept lot 2 with the probability of only 5% (95% probability of rejection). In other words, the producer's risk (α) would be 5% and the owner's risk (β) would be 5%.

Fig. 2 shows the underlying populations of the film thickness for the two lots. In Fig. 2, μ_1 and μ_2 are the average film thicknesses for the population of lots 1 and 2, respectively. After samples of the size n are taken from lot 1, the sample mean of lot 1 noted is $\bar{X}_1$. By central limit theory, the unbiased estimate of $\bar{X}_1$ is μ_1, and the sample mean standard deviation is $\sigma(\bar{X}_1)/\sqrt{n}$. Likewise, the sample taken from lot 2 would have $\bar{X}_2$ equal to μ_2 and the sample standard mean deviation of $\sigma(\bar{X}_2)\sqrt{n}$ (Fig. 2).

In Fig. 2, W is the criteria used to check against the sample mean, $\bar{X}$, for making the decision of accepting or rejecting the lot. To be more specific, if $\bar{X} \geq W$, the lot is accepted. If $\bar{X} < W$, the lot is rejected. Now the questions will be (1) What should the sample size n be? and (2) How will be find the value of the parameter W?

To derive the equations for n and W, an intermediate parameter k connecting n and W is introduced. The notations used are as follows: n = desired sample size; k = an intermediate parameter; W = decision parameter for checking with the sample mean; $Z(\ )$ = value found in the normal distribution table; SD = estimated standard deviation obtained from the sample; $\bar{X}$ = sample mean; and L = specified lower limit (in the previous example, L for primer DFT is 2.5 mils).

Equations for k and n are shown as follows [detailed derivation is presented in Burr's (1976) study]:

$$k = -\frac{Z(\alpha)\cdot Z(RQL) + Z(\beta)\cdot Z(AQL)}{Z(\alpha) + Z(\beta)} \quad (2)$$

$$n = \left(\frac{k^2 + 2}{2}\right)\left[\frac{Z(\alpha) + Z(\beta)}{Z(AQL) - Z(RQL)}\right]^2 \quad (3)$$

$$W = L + k\cdot SD \quad (4)$$

To apply this acceptance method, the AQL, RQL, α, and β risks are first defined. By inserting the four parameters into (1) and (2), the required sample size n and the intermediate parameter k will be obtained. After the inspector takes a sample, the sample mean of $\bar{X}$ and standard deviation of SD are calculated. By applying the L, k, and SD into (3), the decision parameter W is obtained. Finally, the inspector can check $\bar{X}$ against W. If $\bar{X} \geq W$, the lot is accepted. If $\bar{X} < W$, the lot is rejected. The processes for applying this method are summarized in Fig. 3.

Example

Given that the lower limit (L) for the primer DFT is 2.5 mils (6.35×10^{-2} mm), the agency would like to take the risks at 5% level for both sides (owners and contractors), and design an acceptance-sampling plan to comply with the following conditions: AQL = 10% = 0.1 with α = 5%; and RQL = 40% = 0.4 with β = 5%. First, by checking the normal distribution table, the following values are obtained. Z(AQL) = Z(0.1) = −1.29; Z(RQL) = Z(0.4) = −0.26; and Z(α) = Z(β) = Z(0.05) = −1.64.

Inserting these values into (1) and (2) provides the following:

$$k = -\frac{Z(\alpha)\cdot Z(RQL) + Z(\beta)\cdot Z(AQL)}{Z(\alpha) + Z(\beta)}$$

$$= -\frac{-1.64\cdot(-0.26) + -1.64\cdot(-1.29)}{-1.64 + (-1.64)} = 0.775 \quad (5)$$

$$n = \left(\frac{k^2 + 2}{2}\right)\left[\frac{Z(\alpha) + Z(\beta)}{Z(AQL) - Z(RQL)}\right]^2$$

$$= \left(\frac{0.775^2 + 2}{2}\right)\left[\frac{-1.64 + (-1.64)}{-1.29 - (-10.26)}\right)^2 = 13.18 \quad (6)$$

The sample size, n = 13.18, must be a whole number, and is therefore rounded up to 14. With the previous information, the agency can specify the acceptance-sampling plan for primer film thickness as follows:

> The lower limit (L) for the dry film thickness of primer is 2.5 mils. In each lot, a sample size of 14 will be taken. The obtained sample mean is $\bar{X}$; the standard deviation is SD. $W = 2.5 + 0.775SD$. If $\bar{X} > W$, then accept the lot; if $\bar{X} \leq W$, then reject the lot.

If the inspector takes the sample with size n = 14, and gets $\bar{X}$ = 3.7 mils (9.40×10^{-2} mm), and SD = 1.5 mils (3.81×10^{-2} mm), then

$$W = L + k\cdot SD = 2.5 + 0.775\cdot 1.5 = 3.665 \quad (7)$$

The acceptance parameter W = 3.665, and the acceptance rules are as follows: If $\bar{X} > W$ = 3.665, then accept the lot; if $\bar{X} \leq W$ = 3.665, then reject the lot. Here, assume that the sample mean $\bar{X}$ is 3.7 mils (9.40×10^{-2} mm), which is larger than W = 3.665. Thus, the inspector should accept this lot.

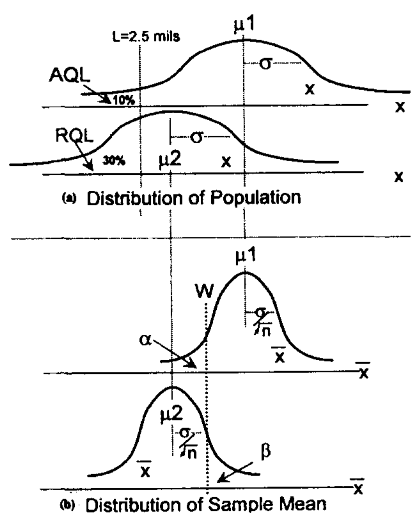

FIG. 2. Distributions: (a) Population; (b) Sample Means

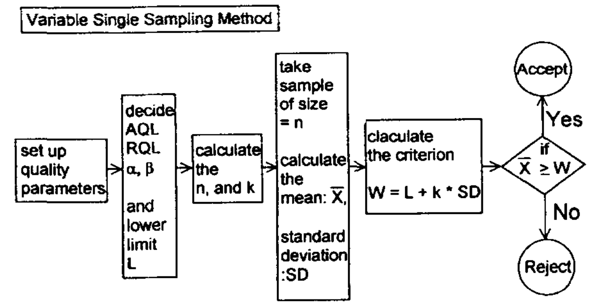

FIG. 3. Processes of Variable-Single-Sampling Method

Quality-Index-Sampling (QIS) Method

Some highway agencies have adopted statistical acceptance specifications in the pavement area (Weed 1989). In these specifications, the sampling methods, sample sizes, and the rules for making acceptance decisions are defined. Among the different acceptance-sampling methods, the quality-index-sampling (QIS) method is frequently used.

The QIS method is a simplified statistical approach without the α and β risk control. With this method, an acceptance criterion called allowable percentage of defect (APD) could be prespecified by owners. After taking samples, the quality index described in the previous section is used to estimate the percentage of defect (PD). Then, the estimated PD is compared with the allowable percent of defective (APD) to make an acceptance decision: If $PD \leq APD$, then accept the lot; if $PD > APD$, then reject the lot. The following equations illustrate the applications of this method. The notations here are SD = estimated standard deviation obtained from the sample; $\bar{X}$ = sample mean; and L = specified lower limit. Let

$$Q = \frac{\bar{X} - L}{SD} \quad (8)$$

By checking with a quality-index table, the PD can be easily obtained. Then the PD is then used to check with the specified APD, which is predetermined by owners (Weed 1989): If $PD \leq APD$, then accept the lot; if $PD > APD$, then reject the lot.

The advantage of this method is its ease of use. Also, the sample size can be subjectively defined by the cost of measurement, the facility available, the imposed time constraint, and so forth. However, this method has a serious drawback in that it provides no information for the probability of acceptance for different quality levels. In other words, the α and β risks are not analyzed or controlled. Thus, the owners and contractors have no idea about the probability of acceptance for a particular lot with a given quality level. The processes for applying this method are summarized in Fig. 4.

Example

The owners may consider that a sample size of 10 is a reasonable workload for inspectors; then the agency subjectively sets the sample size n as equal to 10. A possible specification can be:

> The dry film thickness of primer should be no less than 2.5 mils. That is, the lower limit (L) is 2.5 mils. The APD is 20%. In each lot, a sample size 10 will be taken. The sample mean is $\bar{X}$; the estimated standard deviation of the underlying population is SD. To estimate the percentage out of lower limit, the quality index for lower limit is:
>
> $$Q = \frac{\bar{X} - L}{SD} \quad (9)$$
>
> By checking Q with the quality-index table, the estimated percentage defective (PD: percentage out of lower limit L) is obtained: If $PD \leq APD = 20\%$, then accept the lot; if $PD > APD = 20\%$, then reject the lot.

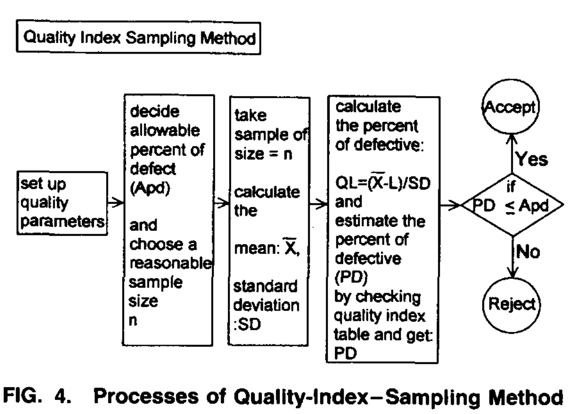

FIG. 4. Processes of Quality-Index-Sampling Method

Assume an inspector takes 10 dry film thickness readings (a sample size of 10) from a lot. Further assume that the sample mean $\bar{X}$ is 3.05 mils (7.75×10^{-2} mm), and the standard deviation of SD is 1.11 mils (2.80×10^{-2} mm). Giving the APD as 20%, the quality index would be:

$$Q = \frac{\bar{X} - L}{\sigma} \approx \frac{\bar{X} - L}{SD} = \frac{3.35 - 2.5}{1.11} = 0.495 \quad (10)$$

From the quality-index table (check at $n = 10$ and $Q = 0.49$), the estimated PD is 31.72%. Because $PD = 31.72\% > APD = 20\%$, this lot will be rejected.

Attribute-Double-Sampling (ADS) Method

The double-sampling plan utilizes two sample sizes along with acceptance-rejection parameters. The notations of the parameters are described as follows.

- n_1 = required sample size in the first sampling.
- n_2 = required sample size in the second sampling.
- c_1 = first acceptable number. If the number of defect (nonconforming items) found in the first sampling is less than or equal to this number (c_1), the lot is accepted.
- r_1 = first rejectable number. If the number of defect found in the first sampling is larger than or equal to this number (r_1), the lot is rejected.
- c_2 = second acceptable number. The parameter is checked when the second sampling is taken. If the total number of defect (including the first and second sampling) is less or equal to this number (c_2), the lot is accepted. Otherwise the lot should be rejected.
- x_1 = number of nonconforming items found in the first sampling.
- x_2 = number of nonconforming items found in the second sampling.

Using the double-sampling plan, the inspectors first take the sample with the size of n_1. If the number of nonconforming measurements (x_1) is equal to or less than the first acceptance number (c_1), the lot is accepted. If the number of nonconforming units is equal to or greater than the first rejection number (r_1), the lot is rejected.

If the nonconforming units fall between c_1 and r_1, a second sample size of n_2 is taken. The number of nonconforming items found in the second sample is x_2. If the total nonconforming items ($x_1 + x_2$) from the first and second sampling ($n_1 + n_2$ measurements) are less than or equal to the second acceptance number c_2, the lot is accepted. Otherwise, the lot should be rejected

To sum up, the procedures of this method are: if $x_1 \leq c_1$, then accept the lot; if $x_1 \geq r_1$, then reject the lot; and if $c_1 < x_1 < r_1$, then a second sample of size n_2 should be taken.

When the second sampling is necessary, continue the following processes: if $x_2 + x_1 \leq c_2$, then accept the lot; and if $x_2 + x_1 > c_2$ then reject the lot.

According to the statistical sampling theory, if the lot size (underlying population) is finite, the hypergeometric distribution should be used to describe the probability of the sampling. In other words, if the lot size is much larger than the sample size, the binomial distribution should be used to calculate the probability of the sampling (Burr 1976).

In accepting construction quality, numerous measurements can possibly be taken from one lot. However, only a few measurements are taken from one lot to define the quality. Compared with the sample size, the lot size (number of the underlying population) is large and can be regarded as unlimited. For this reason, the binomial distribution can be used to develop the sampling theory. The notations used in the equations are PD = underlying percentage of nonconformity (percentage of defect); n = sample size; x = number of nonconforming percentage of defect found in the sample; and $P(\ )$ = binomial distribution.

Given that the product has an underlying percentage of nonconforming PD (percentage of defect), according to the binomial statistics, the probability for getting x nonconforming items under the sample size of n is as follows:

$$P(x) = \frac{n!}{(x!)(n - x)!}\mathrm{PD}^{x}(1 - \mathrm{PD})^{(n-x)} \tag{11}$$

For example, assume that the sample size is 10 (n = 10), and the underlying percentage of defect is 15% (pd = 0.15). By plugging the parameters into (11), the probability of getting three (x = 3) nonconforming measurements will be:

$$P(3) = \frac{10!}{(3!)(10 - 3)!}0.15^{3}(1 - 0.15)^{(10-3)} = 12.989$$

The probability that a lot will be accepted is designated as Pa. The Pa can be accumulated in two conditions: Pa_1 = the probability that lot will be accepted at the first sampling; and Pa_2 = the probability for a lot to be accepted at the second sampling. Pa = probability that the lot will be accepted at the first sampling plus probability that the lot will be accepted at the second sampling; i.e., $Pa = Pa_1 + Pa_2$. The probability that the lot will be accepted at the first sampling can be $Pa_1 = P(x_1 = 0) + P(x_1 = 1) + P(x_1 = 2) + \cdots + P(x_1 = c_1)$; or

$$Pa_1 = P(x_1 \le c_1) \tag{12}$$

To calculate the probability that lot will be accepted in the second sampling, two conditions should happen sequentially; the product is neither accepted nor rejected in the first sampling; and the products are accepted in the second sampling. By multiplying the probabilities of the two conditions, the probability for accepting the product in the second sampling can be represented as follows:

$$\begin{aligned} Pa_2 = {} & p(x_1 = c_1 + 1)\cdot P(x_2 + x_1 \le c_2) \\ & + P(x_1 = c_1 + 2)\cdot P(x_2 + x_1 \le c_2) \\ & + P(x_1 = c_1 + 3)\cdot P(x_2 + x_1 \le c_2) \\ & + P(x_1 = c_1 + 4)\cdot P(x_2 + x_1 \le c_2) \\ & + \cdots + P(x_1 = r_1 - 1)\cdot P(x_2 + x_1 \le c_2) \end{aligned} \tag{13}$$

Simplify this equation as follows:

$$\begin{aligned} Pa_2 = {} & P(x_1 = c_1 + 1)\cdot P(x_2 + c_1 + 1 \le c_2) \\ & + P(x_1 = c_1 + 2)\cdot P(x_2 + c_1 + 2 \le c_2) \\ & + P(x_1 = c_1 + 3)\cdot P(x_2 + c_1 + 3 \le c_2) \\ & + P(x_1 = c_1 + 4)\cdot P(x_2 + c_1 + 4 \le c_2) \\ & + \cdots + P(x_1 = r_1 - 1)\cdot P(x_2 + r_1 - 1 \le c_2) \end{aligned} \tag{14}$$

Then

$$\begin{aligned} Pa_2 = {} & P(x_1 = c_1 + 1)\cdot P[x_2 \le c_2 - (c_1 + 1)] \\ & + P(x_1 = c_1 + 2)\cdot P[x_2 \le c_2 - (c_1 + 2)] \\ & + P(x_1 = c_1 + 3)\cdot P[x_2 \le c_2 - (c_1 + 3)] \\ & + P(x_1 = c_1 + 4)\cdot P[x_2 \le c_2 - (c_1 + 4)] \\ & + \cdots + P(x_1 = r_1 - 1)\cdot P[x_2 \le c_2 - (r_1 - 1)] \end{aligned} \tag{15}$$

$$Pa_2 = \sum_{i=c_1+1}^{r_1-1} P(x_1 = i)\cdot P[x_2 \le (c_2 - i)] \tag{16}$$

The accumulated probability that products will be accepted is as follows: Pa = probability that the lot will be accepted at the first sampling plus the probability that the lot will be accepted at the second sampling; i.e., $Pa = Pa_1 + Pa_2$

$$\begin{aligned} Pa = {} & Pa_1 + Pa_2 = P(x_1 \le c_1) \\ & + \sum_{i=c_1+1}^{r_1-1} P(x_1 = i)\cdot P[x_2 \le (c_2 - i)] \end{aligned} \tag{17}$$

Thus, $Pa = F(n_1, n_2, c_1, r_1, c_2, PD)$: Pa = a function of six parameters: n_1, n_2, c_1, r_1, c_2, and PD (Wadsworth et al. 1986). The processes for applying this method are summarized in Fig. 5.

Example

Assume an acceptance plan has the following parameters: n_1 = 10 = required sample size in the first sampling; n_2 = 10 = required sample size in the second sampling; c_1 = 1 = first acceptable number; r_1 = 3 = first rejectable number; c_2 = 4 = second acceptable number; x_1 = number of the nonconforming items found in the first sampling; and x_2 = number of the nonconforming items found in the second sampling. All of the possible conditions that the lot will be accepted are as follows: $x_1 = 0$; $x_1 = 1$; $x_1 = 2$ and $x_2 = 0$; $x_1 = 2$ and $x_2 = 1$; and $x_1 = 2$ and $x_2 = 2$.

$$\begin{aligned} Pa = {} & P(x_1 \le 2) + \sum_{i=1+1}^{3-1} P(x_1 = i)\cdot P[x_2 \le (c_2 - i)] \\ = {} & P(x_1 = 0) + P(x_1 = 1) + P(x_1 = 2)\cdot P(x_2 = 0) \\ & + P(x_1 = 2)\cdot P(x_2 = 1) + P(x_1 = 2)\cdot P(x_2 = 2) \end{aligned} \tag{18}$$

where

$$P(0) = \frac{10!}{(0!)(10 - 0)!}\mathrm{PD}^{0}(1 - \mathrm{PD})^{(10-0)}$$

$$P(1) = \frac{10!}{(1!)(10 - 1)!}\mathrm{PD}^{1}(1 - \mathrm{PD})^{(10-1)}$$

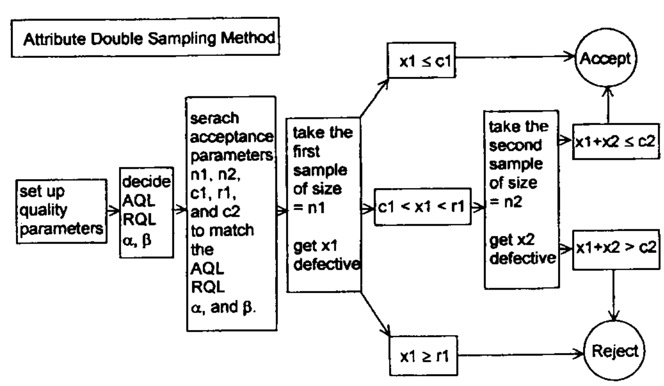

FIG. 5. Processes of Attribute–Double-Sampling Method

$$P(2) = \frac{10!}{(2!)(10-2)!} PD^2(1 - PD)^{(10-2)}$$

When the variable *PD* changes, *Pa* changes correspondingly.

For example, a product has $PD = 6\%$, which is assumed to be the AQL. By plugging the $PD = 6\%$ into (17), $Pa = 95\%$ will be obtained. The associated α risk is 5%, because this product with AQL still has a 5% chance to be rejected. Likewise, a product has $PD = 39\%$, which is assumed to be the RQL. By plugging the $PD = 39\%$ into (17), $Pa = 5\%$ will be obtained. The associated β risk is 5%, because this product with RQL still has 5% chance it will be accepted. With this information, the agency can specify the acceptance plan for primer film as follows:

> The lower limit (L), for the dry film thickness of primer is 2.5 mils. In each lot, the first inspection sample of size 10 should be taken by inspectors, and the numbers of nonconforming items found is designated as x_1: if $x_1 \le c_1 = 1$, then accept the lot; if $x_1 \ge r_1 = 3$, then reject the lot; and if $c_1 < x_1 < r_1$, then a second sample of size 10 should be taken. Nonconforming items found in the second sampling are designated as x_2. So $x_1 + x_2$ nonconformings in the two samples of size $n_1 + n_2$ are found: if $x_2 + x_1 \le c_2 = 2$, then accept the lot; and if $x_2 + x_1 > c_2 = 2$, then reject the lot. The risk-control points are AQL = 6% = 0.06 at α = 5%; and RQL = 39% = 0.39 at β = 5%.

Attribute–Proportion-Sampling (APS) Method

The attribute–proportion-sampling (APS) method is a simplified approach to develop acceptance-sampling plans. The APS is a modification from the error-margin (EM) method. The following section will first discuss the EM method (Neter et al. 1988), and then the APS method will be derived. The notations used are as follows: n = sample size; p = underlying percentage of defective of the lot; and $\mu(\ )$ = expected mean value. By the binomial-statistic theory, the distribution of percentage of defect of the sample is as follows:

$$\text{estimated } \mu(p) = p \qquad (19)$$

$$\text{estimated } \sigma_p = \sqrt{\frac{p(1-p)}{n}} \qquad (20)$$

For (20), see Fig. 6. The underlying percentage of defective can be evaluated within the error margin (h) for the confidence level of $(1 - \alpha)$. That is, the real underlying *PD* should fall within the range of $p \pm h$ with the probability of α (Neter et al. 1988). The binomial distribution shows the following:

$$h = Z(1 - \alpha/2)\sigma_p = Z(1 - \alpha/2)\sqrt{\frac{p(1-p)}{n}} \qquad (21)$$

In the EM method, the error margin (h) is used to design sampling plans. If a sample of size n is taken, the resulting percent of defective is p. With the confidence level $(1 - \alpha)$, the underlying *pd* will fall within the margin of $p \pm h$ (error margin), as follows:

$$p \pm Z(1 - \alpha/2)\sqrt{\frac{p(1-p)}{n}} \qquad (22)$$

Eq. (21) can be also derived to become the following:

$$n = \frac{Z^2(1 - \alpha/2)[p(1-p)]}{h^2} \qquad (23)$$

Eq. (23) can be used to calculate the required sample sizes. At first, the confidence levels $(1 - \alpha)$ and the tolerable error margin (h) can be decided. Use the historical data to predict the underlying percent of defective (p). Once the h, $(1 - \alpha)$, and p are defined, the required sample size n can be obtained with (23).

For example, assume that the owners would like to control the error margin to be 5% ($h = 5\%$) with a confidence level of $(1 - \alpha) = 95\%$. If the historical percentage of defect is 8%, the required sample size is computed as follows:

$$n = \frac{Z^2(1 - a/2)[p(1-p)]}{h^2} = \frac{1.96^2[0.08(1 - 0.08)]}{0.05^2} \approx 113$$

With this approach, the accuracy to estimate the quality is controlled by error margin (h). The smaller the error margin (h) required, the larger the sample size (n) should be. In accepting the quality of finished product, the owners specify the allowable percentage of defect (*APD*) in the contract. By controlling the error margins (h) and confidence level α, the required sample size n can be obtained. If the resultant percentage of defect ($p = x/n$) is smaller than the specified limits, the lot will be accepted; otherwise, it will be rejected.

However, the EM method have two disadvantages. First, the assumption of p in (23) causes errors. The percentage of defect (p) of the underlying population is unknown before inspections. Second, there is no control over the producer's (α) and owner's (β) risks. As mentioned, the risk analyses are important in designing acceptance-sampling plans. The EM method needs to be modified. Therefore, the attribute–proportion-sampling (APS) method is derived. The APS can control the producer's (α) and owner's (β) risks, and needs no assumption of p.

To derive the attribute–proportion–single-sampling plan, two control points are utilized to set the sample size n and decision parameters W. Here, W is used to check against the estimated *PD* to satisfy the following conditions: (1) the products with the AQL have the chance of α to be rejected; and (2) the products with the RQL have the chance of β to be accepted (see Fig. 7).

If a sample size of n is taken and x is the number of nonconforming items found in this sample, the percentage of defect (p) is estimated by x/n. The distribution of percentage defect of the sample is as follows:

$$\text{estimated } \mu(p) = p = x/n \qquad (24)$$

$$\text{estimated } \sigma_p = \sqrt{\frac{p(1-p)}{n}} \qquad (25)$$

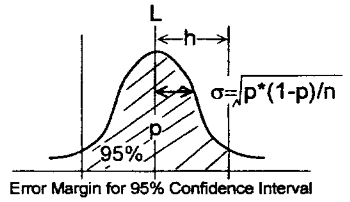

FIG. 6. Error Margin for 95% Confidence Interval

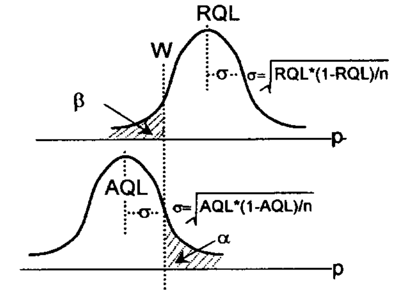

FIG. 7. Proportional Distribution for AQL and RQL

If $p \le W$, then accept; and if $p > W$, then reject. The probability of accepting RQL quality is as follows:

$$\frac{(W - RQL)}{\sqrt{\frac{RQL(1 - RQL)}{n}}} = Z(\beta) \quad (26)$$

The probability of rejecting AQL quality is as follows:

$$\frac{(W - AQL)}{\sqrt{\frac{AQL(1 - AQL)}{n}}} = Z(\alpha) \quad (27)$$

$Z(\ )$ can be checked from the normal distribution table. Solving these two system equations [(26) and (27)] shows the following:

$$n = \left[\frac{Z(\alpha)\sqrt{AQL(1 - AQL)} + Z(\beta)\sqrt{RQL(1 - RQL)}}{(AQL - RQL)}\right]^2 \quad (28)$$

$$W = AQL - Z(\alpha)\sqrt{\frac{AQL(1 - AQL)}{n}} \quad (29)$$

or

$$W = RQL + Z(\beta)\sqrt{\frac{RQL(1 - RQL)}{n}} \quad (30)$$

With (28) and (29), the sample size n and decision parameters W are obtained. The estimated $\mu(p)$ is $p = x/n$: if $p \le W$, then accept; and if $p > W$, then reject. The processes for applying this method are summarized in Fig. 8.

Example

Assume that agency would like to set the following conditions: AQL = 10%; RQL = 30%; control of the producer's risk (α) = 5%; and control of the owner's risk (β) = 5%. Applying (28) gives the following:

$$n = \left[\frac{Z(\alpha)\sqrt{AQL(1 - AQL)} + Z(\beta)\sqrt{RQL(1 - RQL)}}{(AQL - RQL)}\right]^2 \quad (31)$$

The required sample size is as follows:

$$n = \left[\frac{-1.65\sqrt{0.1(1 - 0.1)} + (-1.65)\sqrt{0.3(1 - 0.3)}}{(0.1 - 0.3)}\right]^2 = 30$$

To get the accept/reject decision parameter W, apply (29) as follows:

$$W = AQL - Z(\alpha)\cdot\sqrt{AQL\cdot(1 - AQL)/n}$$
$$= 0.1 - (-1.65)\cdot\sqrt{0.1\cdot(1 - 0.1)/30} = 19.04\%$$

The estimated PD is calculated by $p = x/n$ (x = number of nonconforming items): if $p \le W$ = 19.04%, then accept; and if $p > W$ = 19.04%, then reject. With this information, the agency can specify the contract for primer DFT as follows:

> The dry film thickness of primer should be no less than L, which is 2.5 mils. In each lot, a sample of size 30 will be taken by inspectors. The number of defective from a sample is x. The estimated percent of defective is $p = x/30$: if $p \le W$ = 19.04%, then accept the lot; and if $p > W$ = 19.04%, then reject the lot. The risk-control points are AQL = 10% = 0.10 at α = 5%; and RQL = 30% = 0.30 at β = 5%.

Assume that the inspector takes n = 30 measurements within one lot, and finds x = 3 nonconforming items. The estimated PD is calculated as $p = x/n$ = 3/30 = 10%. Here, p (=10%) $\le W$ (=19.04%), so accept the lot.

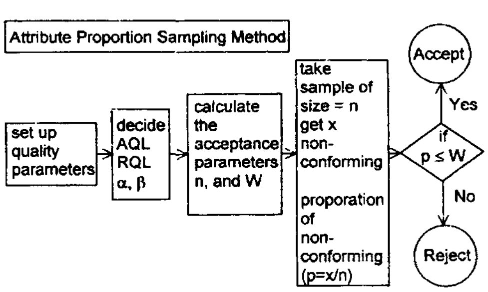

FIG. 8. Processes of Attribute–Proportion-Sampling Method

COMPARISON OF THE FOUR ACCEPTANCE-SAMPLING METHODS

In the previous sections, the four acceptance-sampling methods (ASM) are presented. How can the owners choose the most suitable ASM to develop the QA specification? Unfortunately, there is no easy answer for this question, because none of these acceptance methods will fit all kinds of construction conditions. The four acceptance-sampling methods each have their own advantages and disadvantages. Choosing proper quality-acceptance-sampling methods depends on the characteristics of the construction processes. It may concern the cost of measurement, the required accuracy of decision, the facility available, the imposed time constraints, and the end users' statistics background. The features for each of the ASMs are evaluated, and their advantages and disadvantages are summarized as follows.

VSS Method

Advantage: high accuracy. The VSS method can better utilize the information provided by the data, so that the sample size can be reduced. If the cost for taking one measurement is expensive, the sample size should be reduced and the VSS method is recommended.

Disadvantage: requirement of statistical background. The VSS methods required basic statistical knowledge such as the calculation of standard deviation and the decision parameter W as well as checking the quality index table. However, it is commonly found that construction inspectors do not have enough statistical background to deal with data processing in the VSS method.

QIS Method

Advantage: flexibility in assigning quality level. Often when the owner starts to develop a new QA specification acceptance method, the underlying population of the quality is totally unknown (Blaschke 1989). Therefore, the agency may not be able to specify AQL and RQL based on the unknown quality level. In this case, the QIS method can first be applied, without considering the AQL-α risk, and RQL-β risk. At the initial stage, the agency just specifies the target quality level and defines a reasonable sample size based on past experiences.

Disadvantage: uncontrolled risks. A drawback of QIS is that the owner's (α) and producer's (β) risks are not controlled in this method.

ADS Method

Advantage: ease of use and reduced sample size. The ADS method only requires counting the number of defective items. No statistical calculation is involved in the data process. Also, in the construction industry, many quality factors still rely on inspectors' observation, and cannot be quantified to numerical values. However, in the variable sampling, all quality parameters must be quantified. The ADS, without this constraint, can be used in nonquantified quality parameters. Furthermore, by dividing the sampling phase into two stages, the sample sizes can be reduced.

Disadvantage: attribute sampling provides less accuracy. In general, the variable sampling can produce more accurate estimates than the attribute sampling.

APS Method

Advantage: ease of use and design. The APS is easy to use. The design of the sampling system can be manually calculated without the assistance of a computer.

Disadvantage: less accuracy. Again, the attribute sampling produces a less-accurate estimate than the variable sampling. Also, the APS does not have the advantage of the double sampling, which can reduce the sample size.

CONCLUSION

This paper presents the theories and applications of the four acceptance sampling methods: variable–single-sampling method; quality-index–sampling method; attribute–double-sampling method; and attribute–proportion-sampling method.

The VSS method features the advantage of variable sampling and provides an efficient decision-making mechanism with smaller sample sizes. However, its drawback is that users require basic statistical background to apply this method.

The QIS method is very easy to design and apply. Therefore it is commonly used in the current construction specification. With this method, the sample size can be subjectively defined by the owners. However, this method has a drawback that it does not provide information on the probability of acceptance for different quality levels. In other words, the α and β risks are not controlled. Thus, for a particular lot with a given quality level, both owners and contractors have no idea about the probability of acceptance.

The ADS method reduces the sample size by dividing the sample phase into two stages. To make acceptance decisions, the users only have to count the number of defective measurements. This easy-to-use feature eliminates the need of statistical background for users.

The APS method is a better alternative of the attribute–single-sampling method. Usually, the attribute-sampling method requires a numerical method and a trial-and-error approach to obtain the desired acceptance plan. One advantage of the APS method is that it is easier to calculate than ADS. However, this method usually requires a larger sample size.

When the user designs a quality–acceptance-sampling plan, specifying a rational quality level in the acceptance-sampling plan is essential. Unfortunately, it is often difficult to determine whether or not the designated quality requirement in the specification is realistic or not (Chang and Hsie 1992b). Research revealed that arbitrary acceptance decisions were frequently made. These increased the cost because the contractor has been burned before and thus has to allow contingency for such arbitrariness (Abdum-Nur 1981). Therefore, the AQL-α risk and RQL-β risk need to be defined carefully.

In general, the variable sampling is more accurate than attribute sampling, but the attribute sampling is easy to use. To choose a proper acceptance-sampling method, users need to carefully consider the feature of construction processes and the way of accepting the quality of construction products, including the cost of measurement, the required accuracy of decision, the facility available, the imposed time constraint, and the end users' statistics background.

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