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Selection of the optimal frequencies of viscous damping formulation in nonlinear time-domain site response analysis

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ABSTRACT

Viscous damping is commonly employed in a nonlinear time-domain site response analysis to capture soil damping at small strains. In contrast to the generally accepted concept of the frequency-independent behavior of soil damping, the viscous damping employed as Rayleigh damping is frequency dependent and can overdamp or underdamp wave propagation. This study revisits the issue of selecting the target value of viscous damping frequencies to minimize the effect of frequency-dependent damping. The proposed criterion considers both the site frequency (SF) and frequency characteristics of input motion (e.g., predominant frequency (PF) or mean frequency (MF)) and is more accurate than the widely used protocol in practice. In the Rayleigh damping, the low optimal frequency can be selected as SF but the high optimal frequency should be selected as the maximum between the PF/MF of the input motion and 5SF.

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1. Introduction

One-dimensional site response analysis is routinely performed to assess local site amplification effects during a seismic event [1,2]. The vertical propagation of horizontal shear waves allows the approximation of ground motion, whereas horizontal soil layers represent the site stratigraphy. The solution of the wave propagation equation is performed in either the frequency domain (FD) or time domain (TD). Nonlinear analysis is becoming widely used because it can accurately simulate the nonlinear behavior of the soil and perform effective stress analysis, wherein the development of the seismic pore pressure is modeled. In contrast to laboratory tests that demonstrate the limited influence of the loading frequency [3], nonlinear analysis is limited by the uncontrolled loading frequency dependence of small strain viscous damping.

Viscous damping formulation, which is usually modeled by Rayleigh damping or simplified Rayleigh damping, requires one or two defined frequencies that control the shape and frequency dependence of small strain damping (Fig. 1). For Rayleigh damping, the target damping ratio is matched only at two frequencies, f_0 and f_1 (hereafter called the optimal frequencies). The Rayleigh damping formulation underestimates the damping at frequencies between f_0 and f_1 and overestimates the damping at frequencies lower than f_0 and higher than f_1. The defined frequencies have an important influence on the propagated ground motion [4–8]. Thus, one of the major difficulties in performing nonlinear site response analysis is the selection of formulation frequencies.

In this study, we revisit the aforementioned issue of selecting the optimal frequencies for viscous damping to resolve some of the ambiguities in the current practice. We initially conduct a comprehensive review of previous recommendations. Thereafter, some analyses with bounding cases are performed to assess the recommendations and explore whether they work or not. A solution applicable for general cases is suggested to form the basis of the specifications of viscous damping parameters for most TD codes. The recommendation is verified with a set of analyses that cover a wide range of cases.

2. Selection of optimal frequencies

A few formal protocols are available to guide analysts in selecting the model type and parameters of Rayleigh damping. Most practitioners use simplified or full Rayleigh damping, whereas extended Rayleigh damping [6] is seldom applied in practice. The target damping level ξ is considered the small strain damping (1–5%).

With regard to the optimal frequencies for the full Rayleigh damping, a low frequency f_0 is generally selected as the

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fundamental site frequency (SF), which can be calculated as follows:

$$SF = V_s/4H, \tag{1}$$

where V_s is the elastic shear wave velocity, and H is the thickness of the soil column. A large frequency f_1 is selected as the predominant frequency (PF) or an odd-integer multiple of the fundamental SF. Hudson et al. [4] proposed the use of SF and PF. Hashash and Park [9] showed that favorable matches with FD can be obtained when SF and 8SF are used. However, Park and Hashash [6] found that the conventional guideline in using the first and high mode of the soil column or the PF of the input motion does not always result in a good match with the linear FD solution, particularly for deep soil columns. They concluded that the two significant frequencies should be selected through an iterative process "depending on the input motion". Kwok et al. [7] recommended that when the option of using more than one optimal frequency is available, such as the full Rayleigh damping formulation, this option should be applied in lieu of the simplified Rayleigh damping because significant bias at high frequencies can occur with the latter. The two optimal frequencies in the full Rayleigh damping formulation should be set to SF and 5SF. Their recommendation is based on three selected sites ($SF = =0.45$, 1.06, and 6.4 Hz) that represent a wide range of site conditions. However, the control motion for the evaluation is only one broadband synthetic acceleration history calculated for an outcropping rock site condition.

Phillips and Hashash [10] introduced an approach to constructing a frequency-independent viscous damping matrix and implemented it in DEESPOIL [11]. Rathje and Kottke [12] reported that the frequency-independent damping improves the agreement between TD and FD linear analysis, but TD analysis still present approximately 5–15% underestimation relative to the FD method at frequencies greater than approximately 5 Hz. A potential reason for this difference, according to their explanation, is the time stepping method used in TD analysis (Newmark β method with $\beta = =0.25$), which introduces frequency shortening [13]. However, the transfer function (TF) by TD analyses with frequency-independent damping not only shows frequency shortening but also exhibits additional amplitude decay at a high frequency compared with that by FD analyses. This result indicates that the proposed frequency-independent viscous damping matrix is still frequency independent. The difference between TD and FD results increases with an increase in the small strain damping ratio.

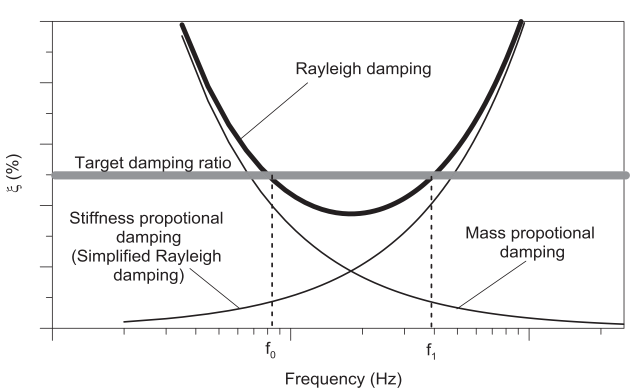

Fig. 1. Simplified and full Rayleigh damping.

The damping parameters should be selected through an iterative process depending on the characteristics of input motion. Thus, the FD and TD elastic solutions can match within a reasonable degree of tolerance over the frequency range of interest. The procedure has been implemented through a user interface in the code DEEPSOIL [11] but is unavailable for other codes. The frequency-independent viscous damping [10] employed in DEEPSOIL still exhibits a frequency-dependent behavior and has not been implemented in any 2D and 3D finite element/finite difference analysis programs. Therefore, the recommendation by Kowk et al. [7] has been mostly adopted by later analyses (e.g., [8,12]) because of its simplicity. However, as discussed previsouly, their guideline neglects the influence of input motion on optimal frequencies.

3. Analysis procedure

3.1. Evaluation approach

A series of analyses is performed to examine the selection of the frequencies/modes of the Rayleigh damping formulations on the site response analysis and to illustrate how the frequency-dependent Rayleigh damping affects the analysis results. Evaluation is performed by comparing the results of the linear TD analyses by using alternative specifications of viscous damping with the exact solution from the linear FD analyses. The FD analyses are exact because of the use of linear soil properties and frequency-independent damping. DEEPSOIL V5.1 [11], which is capable of performing TD and FD analyses, is adopted for all analyses.

3.2. Analysis cases

Three analysis cases listed in Table 1 are performed to investigate how the motion characteristics (as indicated by PF or mean frequency (MF) [14]) coupled with SF affect the optimal frequencies/modes in the Rayleigh damping.

Case 1. Single-frequency motion: One harmonic motion ($PF = =MF = =5$ Hz) is propagated through 50 and 500 m constant V_s profiles.

Case 2. PF and MF of broadband motion lower than SF: One broadband motions of the strong event ($M > >7.0$) recorded at long distances ($R > >100$ km) is propagated through the 30 m constant V_s.

Case 3. PF and MF of broadband motion higher than SF: One motion of the moderate event ($M = = \sim 6.5$) recorded at short

Table 1
Input motions and soil columns for three analysis cases.

Case	Motion						Soil column			
	Type	Event	Station	PF (Hz)	MF (Hz)	PGA (g)	Thickness (m)	V_s (m/s)	ξ (%)	SF (Hz)
1	Harmonic	–	–	5	5	0.3	50	450	1	2.25
							500	450	1	0.23
2	Broadband	Chi-Chi	TAP090	0.86	0.95	0.13	30	250	3.5	2.10
3	Broadband	Northridge	LA00	2.62	2.59	0.39	1000	450	5	0.11

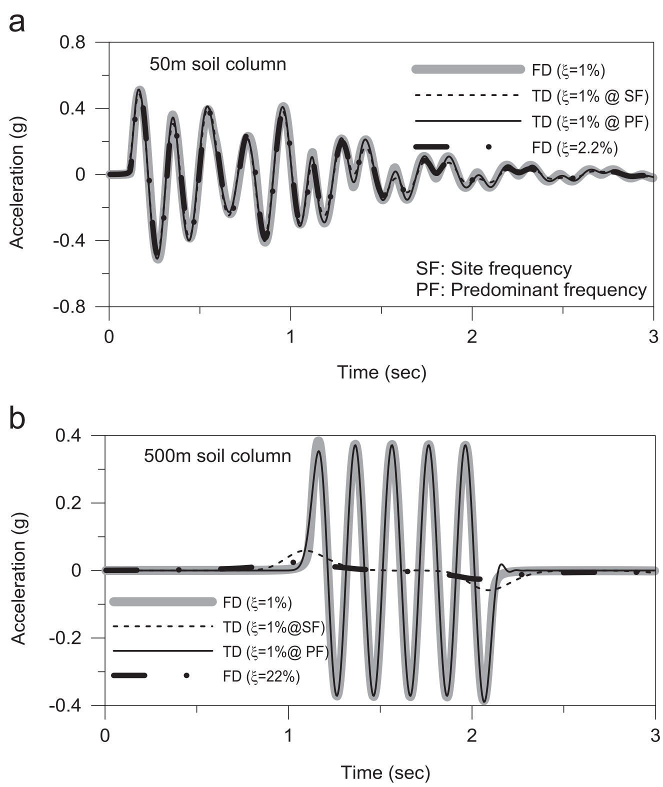

Fig. 2. Computed surface ground motion, linear FD, and TD site response in Case 1.

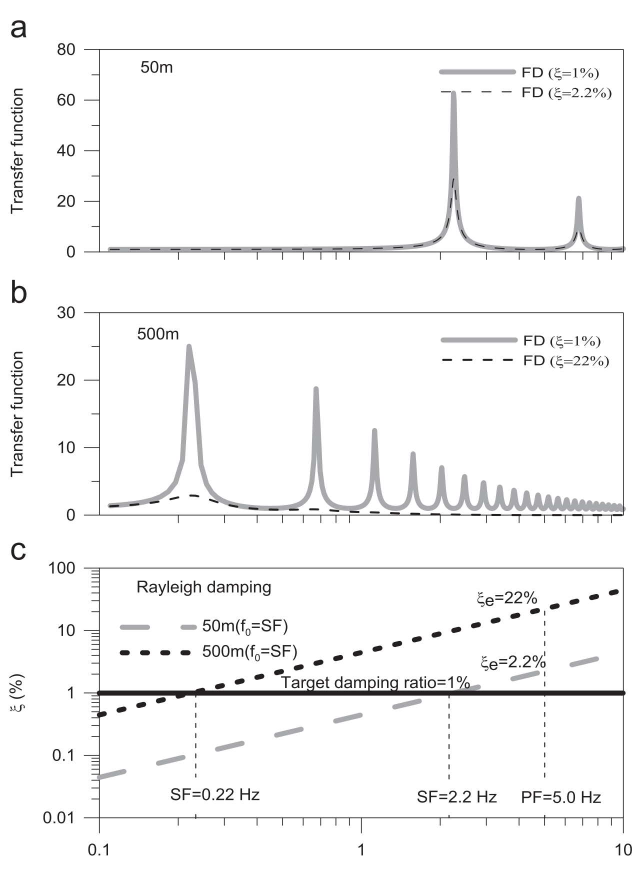

Fig. 3. TFs and effective damping ratio in Case 1.

distances ($R < < 10$ km) is propagated through the 1000 m constant V_s.

The soil column of 30 m is selected because this depth is usually used in site classification, whereas 1000 m soil deposits are encountered in deep basins (e.g., Los Angeles and Taipei). These soil columns with relatively high and low site frequencies stand for two boundary cases that engineers can possibly encounter in site response analysis.

4. Analysis results and discussion

4.1. Case 1 analysis

This analysis case is the same as that performed by Hashash and Park [5]. However, the optimal frequency f_0 selected for simplified Rayleigh damping (one selected frequency only) herein is either the SF or PF of the input motion (i.e., 5 Hz) rather than the SF only.

Fig. 2 compares the surface response calculated by using different optimal frequencies with a target damping ratio of 1%. The use of SF underestimates the surface response compared with the FD analysis, particularly for soil profiles with 500 m thickness. This underestimation is because the use of the simplified Rayleigh damping with SF produces a high damping at 5 Hz, which is the PF of the harmonic motion. On the contrary, the TD analyses that use the PF of the input motion as the optimal frequency consistently match the FD analysis regardless of the soil profile because the damping matches the target damping at 5 Hz. This simple exercise indicates that the use of PF (or MF) is better than SF. This finding differs from the generally accepted concept of using SF as the optimal frequency. The use of simplified Rayleigh damping can also provide an exact response if the optimal frequency is appropriately selected; this result has not been reported by previous studies (e.g., [7]). However, such a condition is only applicable for propagating harmonic motions.

Fig. 3(a) and (b) shows the TFs, and Fig. 3(c) illustrates the effective damping ξ_e introduced by the frequency-dependent Rayleigh damping. By taking the 50 m column as an example, the effective damping at PF or MF (5 Hz) is 2.2% if the optimal frequency is selected as SF (2.2 Hz) (Fig. 3(c)). The TF of the FD analysis by using 2.2% damping is plotted in Fig. 3(a) for comparison. This TF is slightly less than the TF with 1% damping at 5 Hz. Nevertheless, the FD analysis by using 2.2% effective damping and the corresponding TF can result in the same surface response as that estimated by the TD analysis using Rayleigh damping (Fig. 2(a)). Therefore, the effect of the Rayleigh damping can be visualized through the use of the "effective" TF, which is constructed by applying an effective damping that corresponds to the frequency of interest. A similar analysis can be performed for the 500 m column case. Likewise, the FD analysis with 22% effective damping (Fig. 3(c)) and the corresponding TF (Fig. 3(b)) can result in the same surface response as that estimated by the TD analysis by using Rayleigh damping (Fig. 2(b)).

4.2. Case 2 analysis

Fig. 4 shows the calculated 5% damped surface acceleration response spectra, TFs, Fourier spectrum of the input motion, and effective damping curve. Fig. 4(a) shows that the surface responses from the TD analyses by using SF and 5SF agree well with the FD analysis. Even though the PF or MF of the input motion is lower than the SF (Fig. 4(c) and (d)) and selecting the SF and 5SF can result in high damping at the PF and MF (i.e., 7% effective damping versus 3.5% target damping) (Fig. 4(d)), the calculated surface

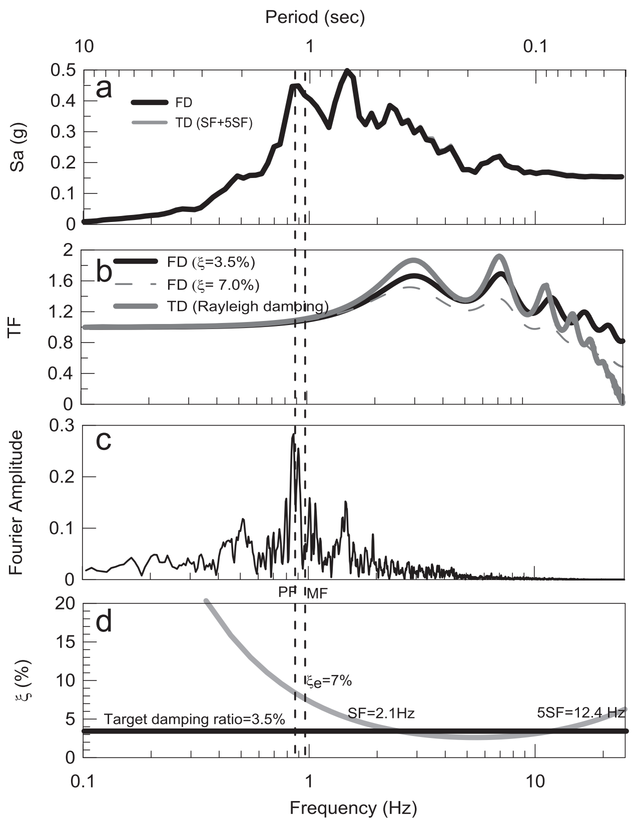

Fig. 4. Surface response spectrum, TF, Fourier spectrum of the input motion, and effective damping ratio of Case 2.

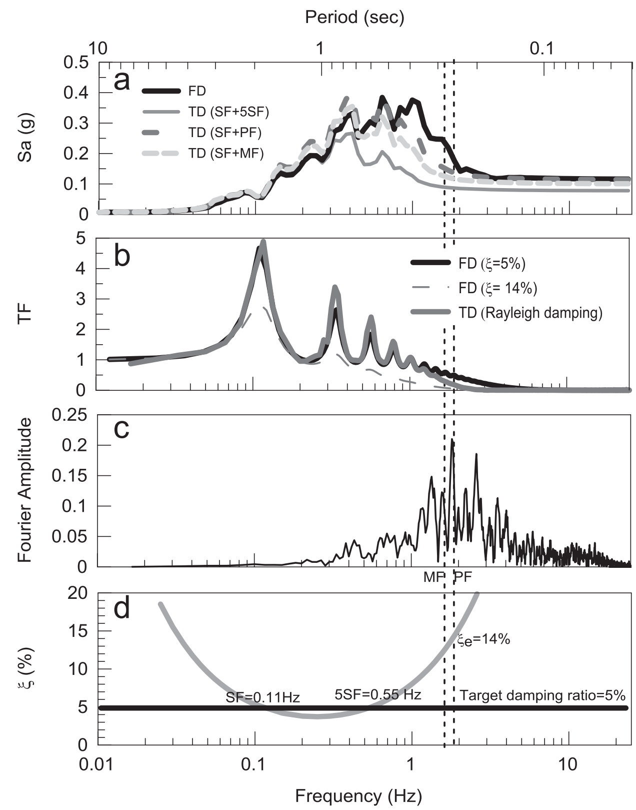

Fig. 5. Surface response spectrum, TF, Fourier spectrum of the input motion, and effective damping ratio of Case 3.

responses are still similar. This result can be explained through the effective TF concept. Fig. 4(b) shows the TF with 7% damping for comparison. Although these TFs differ at SF, the difference at PF is minor because the amplitudes of the TFs monotonically converge toward unity at zero frequency. Therefore, when the PF or MF is lower than the SF, the effect of the frequency-dependent Rayleigh damping is small if SF is selected as one of the optimal frequencies. Thus, the relative difference between TD and FD is minor (within a range of 10%).

4.3. Case 3 analysis

Fig. 5 shows the calculated 5% damped surface acceleration response spectra, TFs, Fourier spectrum of the input motion, and effective damping curve. Fig. 5(a) shows that the surface response calculated by the TD analysis by SF and 5SF is lower than that by FD analysis. The TD analysis with the first and fifth modes can potentially underestimate the surface response because SF is lower than PF or MF. Selecting SF and 5SF can still result in high damping at the PF (14% effective damping versus 5% target damping, Fig. 5(d)). The effective TF with 14% damping around the PF of the motion is similar to that produced by the Rayleigh damping and is lower than the exact TF (Fig. 5(b)). Thus, most of the components of the input motion near the PF or MF (Fig. 5(c)) can be over-damped when propagated through the soil column. In this case, the PF or MF should be selected as one of the optimal frequencies for Rayleigh damping. Fig. 5(a) shows that the use of SF and PF/MF can significantly improve the analysis results compared with the use of SF and 5SF. The use of SF and PF shows a good match with the FD result at periods less than 5 s, whereas the use of SF and MF compares favorably at periods greater than 5 s.

Table 2
Site profiles for verification.

Profile name	Location	Thickness (m)	Bedrock V_s (m/s)	SF (Hz)
P1	Busan, Korea	52	800	0.89
P2	Treasure Island, USA	88	2500	0.74
P3	Anchorage, USA	155	762	0.67
P4	Anchorage, USA	180	732	0.56
P5	Mississippi Embayment, USA	406	3000	0.38
P6	Mississippi Embayment, USA	1000	3000	0.19

Table 3
Input ground motion used in site response analyses.

Ground motion name	Earthquake event	Station name	PGA (g)	PF (Hz)	MF (Hz)
TH1	Loma Prieta, USA	Yerba Buena Island	0.067	0.70	1.17
TH2	Nahanni, Canada	NWT Station #3	0.148	16.06	6.78
TH3	Synthetic motion	–	0.154	2.30	2.00
TH4	Miyagi-Oki, Japan	Ofunato	0.227	3.31	3.74
TH5	Northridge, USA	University Hospital	0.263	2.93	3.23
TH6	Loma Prieta, USA	Gilroy Station 1	0.442	2.69	2.54

5. Discussion

Cases 1–3 demonstrate that the selection of the optimal frequency should not only depend on the SF of the soil column but also on the frequency content of the input motion. In addition to the SF suggested by previous studies, the PF or MF of the ground motions should be considered. When propagating a harmonic motion (Case 1), the PF of the input motion, which is identical to the MF of the input motion, should be selected as the optimal frequency regardless of the thickness of the soil column. When propagating a broadband motion, selecting the first and fifth modes of the soil column as the optimal frequencies is applicable for the shallow soil column. For the deep soil column, where the SF is lower than the PF or MF of the input motion, we recommend selecting the SF and PF or MF of the input motion as the optimal frequencies. Hence, the optimal frequencies should be selected as the maximum between the PF/MF of the input motion and 5SF.

6. Verification of the recommedation

A set of linear site response analyses is performed to evaluate the effectiveness of the recommended optimal frequency by using six soil profiles and six ground motions. The details of the profiles, ranging from 52 m to 1000 m in thickness, are given in Table 2. The characteristics of these motions are listed in Table 3.

The optimal frequencies of the viscous damping formulation are first obtained by the iterative procedure outlined in Park and Hashash [6]. SF is used for f_0, and f_1 is chosen by trial-and-error so that the TD matches most favorably with FD. f_1 is selected such that the averaged difference between TD and FD between periods 0.025 and 4.0 s (the same period range used to calculate the mean period [14]) is the lowest. The frequencies f_1 selected by trial-and-error, considered to be "correct", are compared with those obtained by the recommended procedure outlined in the previous section in Fig. 6.

In Fig. 6a, the f_1 selected by trial-and-error is compared with 5SF. The use of 5SF causes pronounced deviations from the optimal

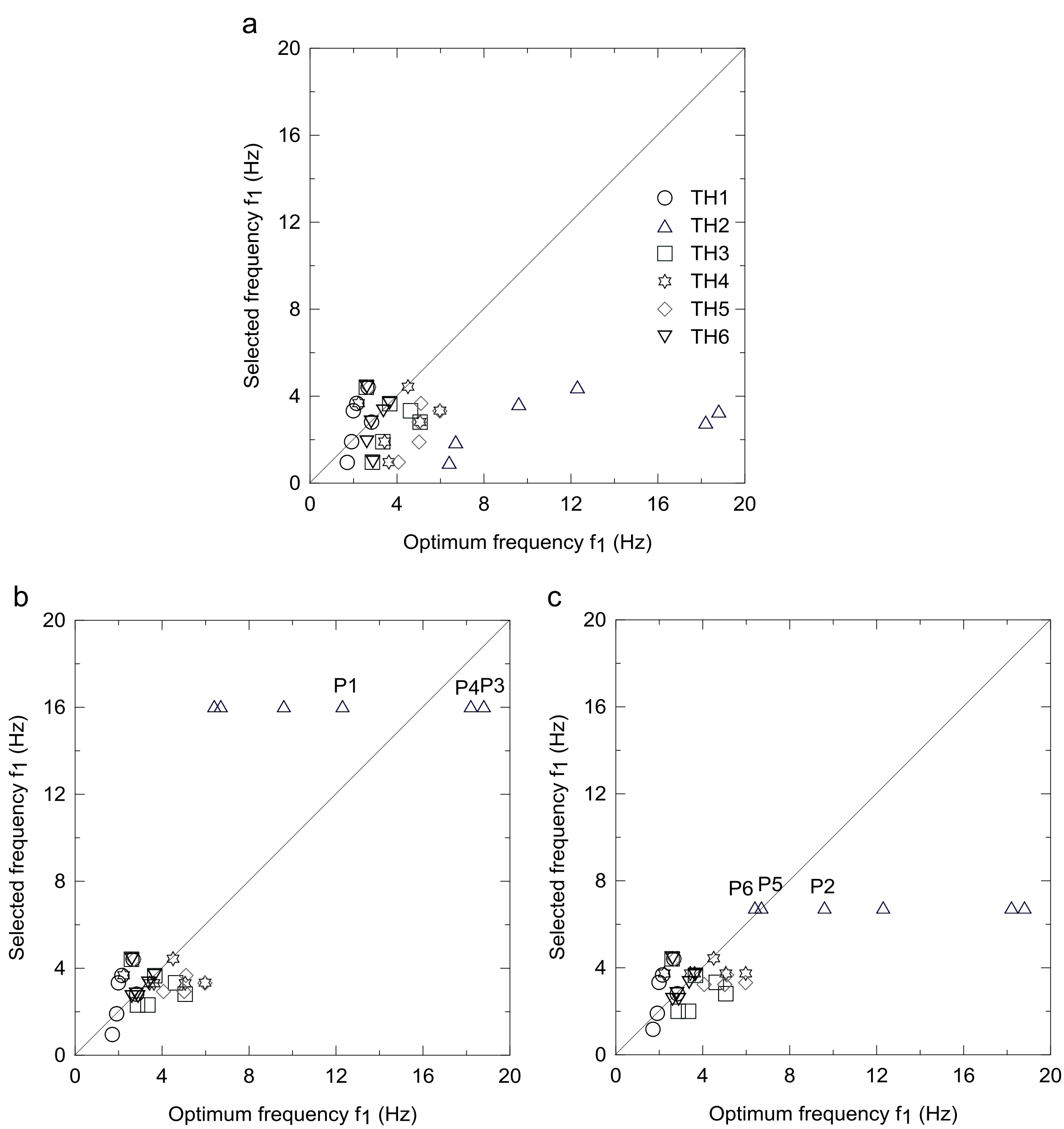

Fig. 6. Comparison of optimal and selected frequencies. (a) $f_1 = = 5SF$ (b) $f_1 = = \max(5SF, PF)$ (c) $f_1 = = \max(5SF, MF)$.

frequencies selected by trial-and-error. By contrast, when the criterion of the maximum between 5SF and PF or between 5SF and MF is used (Fig. 6(b) and (c), respectively), the selected f_1 by the recommendation procedure shows enhanced fits with the correct frequencies. Owing to the insignificant difference between PF and MF, the use of either PF or MF results in an acceptable match with the optimal frequencies, except for TH2. TH2 is a special case because its PF is as high as 16.06 Hz, which is rarely observed for a typical ground motion. The MF for TH2 is significantly lower than the PF at 6.78 Hz, thus indicating that the frequency contents of the motion are evenly distributed in a wide frequency range. When TH2 is analyzed, the use of MF shows good matches for profiles P2, P5, and P6, whereas the use of PF is appropriate for the other profiles. For motions that show significant differences in PF and MF, the optimal f_1 is difficult to determine. However, either PF or MF provides a good estimate of the optimal frequency. If a conservative estimate of the propagated motion is acceptable, a large frequency between PF and MF can be applied. Otherwise, a comparison with FD analysis is required to estimate the optimal frequency accurately. The series of analyses demonstrates that the recommended procedure of using 5SF and PF/MF to select the frequencies is more reliable than using 5SF.

7. Conclusion

Viscous damping is used in the nonlinear TD site response analysis to capture the soil damping at small strains. The approximation of the Rayleigh damping is commonly employed. In contrast to the generally accepted concept of the frequency-independent behavior of soil damping, the Rayleigh damping is frequency dependent and can overdamp or underdamp the wave propagation. This study revisited the issue of selecting the target value of the viscous damping frequencies to minimize the effect of the frequency-dependent damping.

On the basis of the insight discussion of three analysis cases, we concluded that the PF or MF of the ground motions should be considered in addition to the SF suggested by previous studies when selecting the target value of the viscous damping frequencies. When propagating a harmonic motion, the PF or MF of the input motion should be selected as the optimal frequency regardless of the soil column thickness. When propagating broadband motion, selecting the SF and 5SF as the optimal frequencies is applicable for shallow soil column. For the deep soil column, where the SF is lower than the PF of the input motion, we recommended selecting the SF and PF/MF of the input motion as the optimal frequencies. Hence, the second optimal frequencies should be selected as the maximum between the PF/MF of the input motion and the fifth modes of the soil column. For most ground motions, the difference between PF and MF is insignificant. The recommended selection criterion has been verified by a set of analyses. The selected frequencies agree well with those estimated from the iterative and trial-and-error-based scheme. These frequencies will be a suitable starting point for most codes and can be further refined if an iterative process that matches the linear FD and TD solutions is available.

Acknowledgments

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