A coordinate vector correction method to improve the traditional affine transformation graphic digitised cadastral map

S.-P. Kao, C.-P. Tu* and C.-N. Chen

Parameter coordinate transformation regards the whole transformation region under the same condition for coordinate adjustment. However, the practical errors and ideal corrected values are not 'individually' discussed. The proposed coordinate vector correction method uses 'area' and 'point' to further correct individual coordinates in order to amend the blind spots resulting from overall coordinate transformation. The experimental areas in this study were five graphic area sectors in Taichung City. The graphic digitised TWD67 coordinate system was converted using affine transformation into the TWD97 coordinate system. The coordinate vector correction method was used to correct various boundary point coordinates. The results were compared with affine transformation results and analysed. According to the data obtained from the experimental areas, when the traditional affine transformation was used, the root-mean-square error value of the converted and registered areas could be reduced by 6–69% by using the proposed coordinate vector correction method. The quantity of land parcels exceeding the margin of error (the margin of error adopted in this paper is $0{\cdot}2\sqrt{A}+0{\cdot}0003A$, where A is the total area of the land parcel in m^2 units) decreased by 26–35%. As for the boundary point displacement, the average boundary point offset of the areas was 0·3–4·2 cm. The positions of boundary points could be corrected by small amounts using the proposed coordinate vector correction method, so that the discrepancies between the area formed by boundary points and the registered area could be effectively solved.

Keywords: Coordinate transformation, Land revision, Graphic area, Coordinate vector correction method

Introduction

Cadastral areas can generally be classified into four types, the actual area on the earth surface, the registered area with legal restriction, the measurable map area on the cadastral map and the existing area obtained from field survey [**8**]. The actual area on the Earth's surface is an unobtainable true value. The area and coordinates from field survey are supposed to be the closest to the actual area. The map area on the cadastral map is the most original cadastral record information. The registered area influences the owner's rights most directly. Theoretically, the four types of area should be consistent or have only slight differences; however, the actual conditions are always not so [**3**].

The cadastral map of Taiwan has multiple coordinate systems and the differences in coordinate systems often cause many problems. The government is in the process of converting the cadastral data from the cadastral coordinate system or TWD67 system into the TWD97 coordinate system [**1**].

The current graphic cadastral map processing method digitises the graphic cadastral map, and retains its geometric conditions and digital graphical information, using the TWD67 and TWD97 coordinate control points to simultaneously determine the coordinate transformation parameters and transform the boundary points of the whole area into the new coordinate system (TWD97). However, after complete coordinate transformation, the map area is always different from the registered area, which damages the proprietors' rights and interests. The coordinate vector correction method is used to translate the boundary points in minute quantities after the overall parameter transformation, so that the map area coincides with or further approaches the registered area (This paper uses registered area as the target value for calculation).

The whole region is generally regarded under the same condition for coordinate transformation [**2**], such as the Helmert transform, affine transform and projection transform, in the hope of using one mathematical model to solve the global error. However, after global

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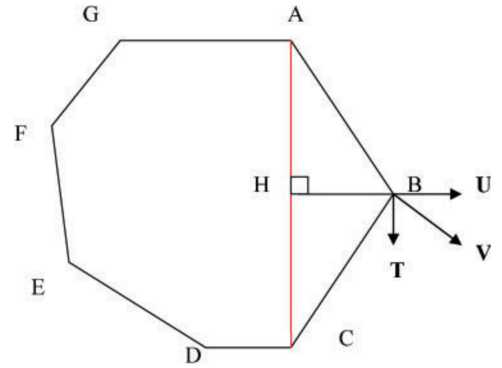


1 Schematic diagram of polygon and triangle and *n*-1-sided shape

coordinate transformation, the random error, map stretching, digitisation and measurement errors cannot be completely eliminated using one mathematical mode. The error propagation, such as instrumental error, plane table setting error and drafting error should also be considered. The standard boundary point error in graphic cadastral map digitisation result is $\pm 0{\cdot}388$ to $\pm 0{\cdot}417$ mm on map [**4**]. In terms of the 1/500 graphic cadastral map in this study, the field boundary point error limit is $\pm 19{\cdot}4$ to $\pm 20{\cdot}9$ cm. There is considerable room for adjustment in the boundary point error limit of the graphic cadastral map.

This study therefore proposes a coordinate vector correction method using 'point' and 'area' displacement as the main correction targets. The proposed method can solve the out-of-tolerance problem in a registered map or existing area and convert that area more extensively and effectively.

Background theory

Coordinate vector correction method

The fundamental coordinate vector correction method theory discusses the variance in an area when one vertex of an arbitrary polygon is given vector displacement. Every vertex in this polygon can be deduced using the same method. At this point, the total changed area due to displacement of all vertices should approach the actual changed area of the polygon.

This proposed method is based on the theoretical method described below.

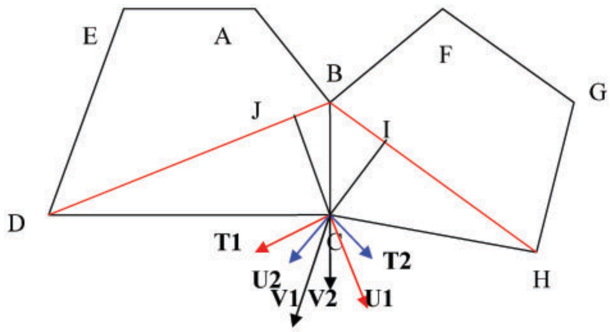
Assume a random *N*-sided shape (e.g. a heptagon) *ABCDEFG*, as shown in Fig. 1, given that any vertex (e.g. point *B*) and two adjacent vertices can form a ΔABC, the line segment *AC* can separate the polygon *ABCDEFG* into ΔABC (area A_2) and *N-1*-sided shape ACDEFG (area A_1).

Given that the line segment **HB** is the vertical line segment **AC**, regarding the area variation in the polygon *ABCDEFG* when the point *B* has moved for **V** vector, the area A_1 is fixed because the vertices of polygon *ACDEFG* have not moved and the area A_2 of ΔABC is

$$A_2 = \frac{(\mathbf{AC} \times \mathbf{HB})}{2} \tag{1}$$

After the displacement of point for vector **V** to *B'*, the area is A'_2, which can be expressed as

$$A'_2 = \frac{[\mathbf{AC} \times (\mathbf{HB} + \mathbf{V})]}{2} \tag{2}$$



2 Schematic diagram of two adjacent polygons and their offset vectors V1 and V2

V can be separated into **U** vertical to and **T** parallel to **AC**

Therefore, the above equation can be expressed as

$$A'_2 = \frac{[\mathbf{AC} \times (\mathbf{HB} + \mathbf{U} + \mathbf{T})]}{2}$$

$$= \frac{1}{2}(\mathbf{AC} \times \mathbf{HB} + \mathbf{AC} \times \mathbf{U} + \mathbf{AC} \times \mathbf{T}) \tag{3}$$

$$\because \mathbf{AC} \perp \mathbf{T}, \quad \text{so} \quad \mathbf{AC} \times \mathbf{T} = 0 \tag{4}$$

$$\therefore A'_2 = \frac{1}{2}(\mathbf{AC} \times \mathbf{HB} + \mathbf{AC} \times \mathbf{U}) \tag{5}$$

If dA is area variation

$$dA = A'_2 - A_2 = \frac{\mathbf{AC} \times \mathbf{HB} + \mathbf{AC} \times \mathbf{U}}{2} - \frac{\mathbf{AC} \times \mathbf{HB}}{2} = \frac{\mathbf{AC} \times \mathbf{U}}{2} \tag{6}$$

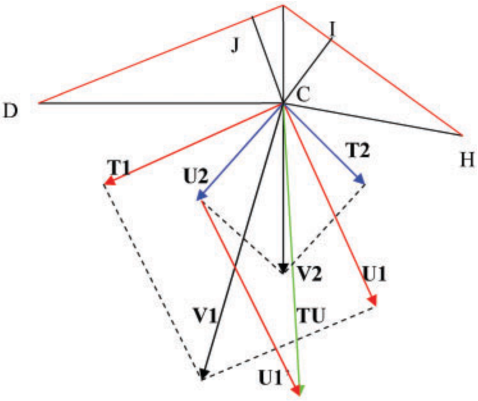
According to the above deduction, any point moves for any vector **V** only the vector **U** perpendicular to the base side would affect that area, whereas the vector **T** parallel to the base side does not, regardless of size. These conditions are applied to two polygons (*see* Fig. 2).

The impact of the displacement of point *C* on the polygon *ABCDE* and the polygon *BFGHC* is similar to that on changes in ΔBCD and ΔBCH areas. As mentioned above, vector **V1** to ΔBCD and **V2** to ΔBCH can be divided into four vectors, namely **U1** perpendicular to **BD**, **T1** parallel to **BD**, **U2** perpendicular to **BH** and **T2** parallel to **BH**. Vectors **T1** and **T2** are parallel to the base line, thus proving to have no effect on the area, so the displacements of vectors **U1** and represent ΔBCD area change (**BD** × **U1**)/2 and ΔBCH area change (**BD** × **U2**)/2 respectively.

Point *C* and **U1**, **U2** part is enlarged for further discussion. **U1** and **U2** are regarded as two stresses that change the area, and vector ***U1'*** is the parallel vector of vector **U1**, thus, the total vector **TU** of this stress is regarded as the optimal offset.

Calculating boundary point displacement the **Uun** vector of each point is obtained before the **Un** vector is calculated. The **Uun** vector is defined as the unit vector in unit lengths of 1 m, normal to the base line and can increase the directions of area. The **Un** vector is the vector unit that can increase the area by 1 m^2 and is normal to the base line. The computing mode is shown below.

If **BD** vector is the base line vector of the triangle vertex to be discussed, the vector is $[X_{DB}, Y_{DB}]$. The



3 Enlarged view of point *C*, vectors U1 and U2

$[X_{DB}, Y_{DB}]$ can be turned 90°clockwise using the plane rotation matrix [7] (if clockwise rotation 90° is the direction that can enlarge the area, otherwise counter clockwise rotation 90°), and the **Uun** vector can be expressed as

$$\mathrm{Uun}=\begin{bmatrix}\cos 90^\circ & \sin 90^\circ\\ -\sin 90^\circ & \cos 90^\circ\end{bmatrix}\begin{bmatrix}X_{DB}\\ Y_{DB}\end{bmatrix}\Big/\left[(X_{DB})^2+(Y_{DB})^2\right]^{1/2} \quad (7)$$

The unit area vector **Un** can be expressed as

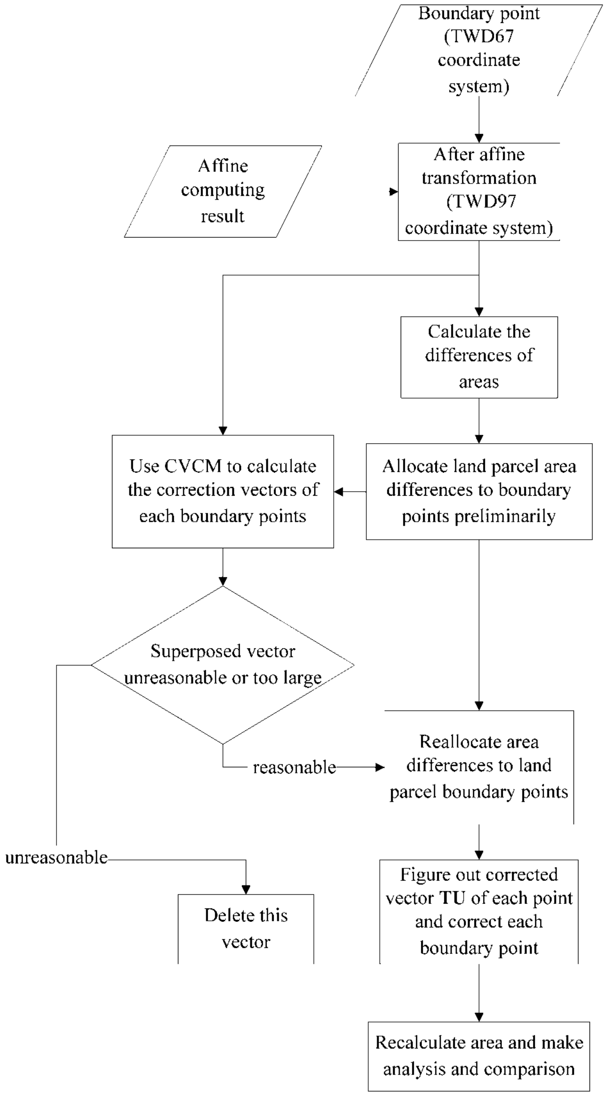
$$\mathrm{Un}=\frac{\mathrm{Uun}}{\mathrm{DB}/2}=2\times\begin{bmatrix}\cos 90^\circ & \sin 90^\circ\\ -\sin 90^\circ & \cos 90^\circ\end{bmatrix}\begin{bmatrix}X_{DB}\\ Y_{DB}\end{bmatrix}\Big/(X_{DB})^2+(Y_{DB})^2 \quad (8)$$

If the same area A is changed

$$\because A=\frac{\text{bottom}\times\text{height}}{2} \qquad A\in C \quad (9)$$

$$\therefore \text{height}\propto\frac{1}{\text{bottom}} \quad (10)$$

According to equation (10), a longer base side indicates smaller height of the same area. When the differences between the converted land parcel area and the registered existing area on the map are allocated, in order to move the boundary points uniformly, the area distribution ratio is proportional to the base side length of the corrected point. The area difference value allocated to each boundary point is multiplied by the **Un** vector of that point, and the product is the translation of the boundary point normal to the base line for the land parcel. If there is adjacent concurrence, the correction should be in the resulting direction as (*see* Fig. 3). If one point is beside three or more points, two of them have the minimum vector result and should be corrected. This procedure requires that all boundary points be calculated to obtain the optimum offset of each boundary point and to correct it. The flow chart of CVCM is in Fig.4.



4 Flow chart of using coordinate vector correction method for coordinate vector correction computation

Experimental method and procedures

Selection of experimental area

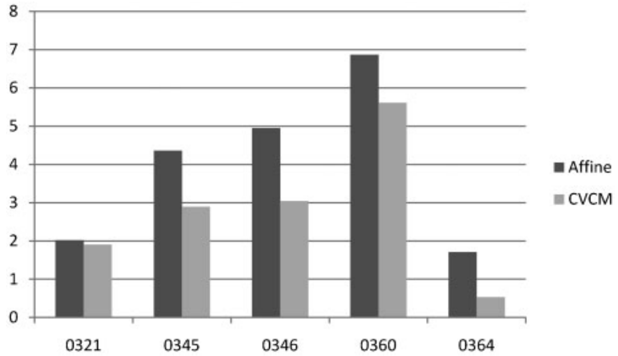
This study selected a sector in the Central District of Taichung City, Taiwan, as the experimental area [5]. The attribute data of the experimental area are shown in Table 1.

Research process

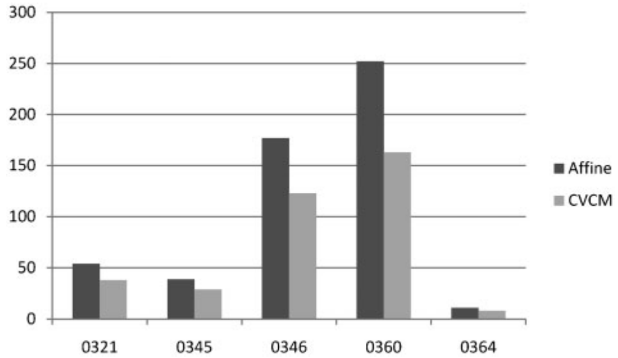
This study performed coordinate transformation and correction on the entire experimental site. Since the transformed common points were control points with TWD67 and TWD97, the transformation was considered overlaying, instead of corrected map sheets stretching and shrinking, thus the affine transformation method would produce better results [6]. The control points were used for traditional affine solving and the boundary points were transformed from the TWD67 coordinate system into the TWD97 coordinate system

Table 1 Attribute data of experimental area

Section code	Measurement mode	Coordinate system	Scale	No. of land parcels	No. of boundary points	Area/ha
0321	Graphical method	TWD67	1/500	2210	5477	45·72
0345	Graphical method	TWD67	1/500	203	477	5·29
0346	Graphical method	TWD67	1/500	709	1690	9·12
0360	Graphical method	TWD67	1/500	1553	3971	35·58
0364	Graphical method	TWD67	1/500	1060	2066	17·50



5 The comparison of the RMSE of each land parcel area after affine transformation and coordinate vector correction method



6 The total number of land parcels and exceeding the maximum permissible error

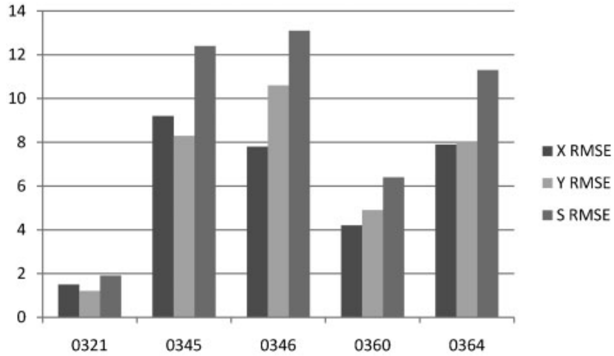
according to the solved affine. The correction of each boundary point was determined and corrected according to the proposed coordinate vector correction method for the affine transformed TWD97 coordinates. The land parcel area and the registered area were then compared and analysed.

Results analysis and comparison

Comparison and analysis of results of traditional coordinate transformation and coordinate vector correction method

After the five experimental sites were processed by affine transformation and corrected by the coordinate vector correction method, the RMSE values of difference between land parcel area and registered area were compared.

The root-mean-square error (RMSE) value of each land parcel area after using the coordinate vector correction method was less than that of the area and the registered area transformed using the affine transformation only (*see* Fig. 5).



7 The analysis chart of the RMSE of correction values of each land parcel area

The number of out-of-tolerance land parcel areas corrected using the traditional affine transformation and coordinate vector correction method was obviously less than that transformed using only the affine method (*see* Fig. 6).

Using the affine transformation and coordinate vector correction method, the boundary point displacement is shown in Table 2.

As shown in Fig. 7 and Table 2, S is the offset distance on XY plane, and MAX is the maximum offset of all boundary points in this area.

According to Table 2, the mean X and Y correction values are close to 0, indicating that the overall coordinates have no offset. The RMSE value of the overall offset is slightly high. In terms of the mean offset and MAX value, there may be outsized values that increase the overall RMSE value.

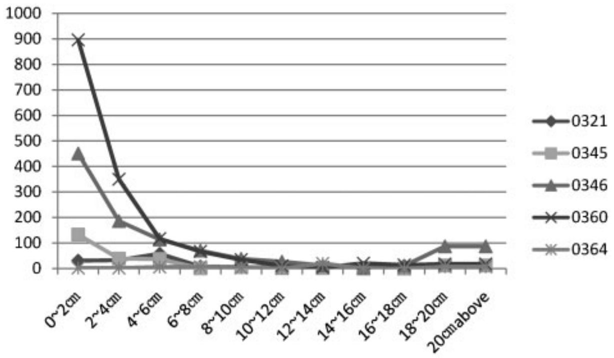
As seen in Fig. 8, the offset of most boundary points is below 8 cm, and the number of offsets above 20 cm is larger than that of 8–2 cm, possibly due to incorrect data. The cause for excessive offset values should be discussed individually.

Conclusions and suggestions

This study presented a coordinate vector correction method that corrects boundary points after applying the traditional affine transformation to reduce the margin of error in adopted areas by >26%. The RMSE of the corrected land parcel area and existing registration area are obviously reduced. According to the boundary point correction results, as shown in Table 2, the corrections were less than ± 8 cm. Boundary points with excessive offset may result from unreasonable or bad geometric data distribution. Further discussion of this problem using case studies is important.

Table 2 Land parcel area RMSE comparison between traditional method and vector correction method

Section	Affine transformation RMSE/m^2	Coordinate vector correction method RMSE/m^2	Improvement ratio/%
0321	2·02	1·90	5·9
0345	4·36	2·89	33·7
0346	4·95	3·04	38·6
0360	6·87	5·61	17·8
0364	1·71	0·53	69·0
Mean value	3·98	2·79	33·0



8 Diagram of the boundary point offset and quantity of each section

The error propagation such as instrumental error, plane table setting error and drafting error, the field boundary point standard error of a 1/500 graphic cadastral map after conversion was in the ±19·4 to ±20·9 cm range. If this range is used as the standard, the number of boundary points exceeding the margin of error adopted was few.

The vector correction method proposed in this study only carries out one-time correction and solution. Future study will focus on repeated solution to improve the precision.

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