Evaluating the Elastic Modulus of Lightweight Aggregate

How-Ji Chen, Tsong Yen, and Kuan-Hung Chen

How-Ji Chen is an associate professor of Civil Engineering at National Chung Hsing University in Taiwan. His research interests include lightweight aggregate concrete and structural analysis.

ACI Member **Tsong Yen** is a professor of National Chung Hsing University in Taiwan. His research interests include lightweight aggregate concrete and high-performance concrete.

Kuan-Hung Chen is a PhD candidate of Civil Engineering at National Chung Hsing University in Taiwan.

ABSTRACT

A constituent model based on the actual distribution of coarse lightweight aggregates in hardened concrete is proposed for estimating the elastic modulus of lightweight aggregate in hardened concrete. A commercially available program (ANSYS) designed to handle both coarse aggregate and mortar matrix as homogeneous isotopic materials was used in this investigation in which the spherical coarse aggregate was assumed positioning at the center of a concrete cylinder. This model is therefore called the "Central Aggregate Model." The numerical model determined the modulus of elasticity of the aggregate based on the equation, Ea = f (Em, Ec, and Va), in which Em (modulus of mortar), Ec (modulus of concrete), and Va (volume fraction of aggregate) are the principal inputs. This paper presents the basic concept of the proposed Central Aggregate Model, test data, and the model's suitability for evaluating the elastic modulus of lightweight aggregate. The Ea equation based on Central Aggregate Model is

$$Ea = 6.3Ec - 5.1Em - 9.1VaEc + 8.7VaEm.$$

Keywords: Elastic modulus, Central Aggregate Model, Concrete, Mortar, Lightweight Aggregate

RESEARCH SIGNIFICANCE

Modulus of elasticity of concrete has been considered as a vital factor in designing concrete structures. To predict the Ec of lightweight concrete effectively, accurate lightweight aggregate E values must first be evaluated. The synthetic lightweight aggregates commonly used in concrete construction possess highly porous interiors and relatively dense exteriors. The non-homogeneity and anisotropy have made it nearly impossible to determine its modulus of elasticity by either a theoretical approach or a directly experimental procedure [1-3]. This study indicates the practicability of using the numerical method to evaluate the elastic modulus of lightweight aggregate based on the Central Aggregate Model.

INTRODUCTION

The stress and strain behaviors within a concrete element under loading greatly depend on the compatibility of both the mortar and coarse aggregate in terms of their modulus of elasticity, Em and Ea respectively. The importance of Ea in this regard is eminent and requires sophisticated engineering analyses. Numerous theoretical methods have been suggested by authors [4] for the assessment of the modulus of elasticity of an aggregate within hardened concrete with the assumption that individual aggregates are homogeneously isotropic and firmly in contact with the surrounding mortar matrix. The binding mechanism involves complicated stress/strain behaviors along the interfaces between the mortar and aggregate particles. Voigt, Reuss, Counto and others proposed parallel connection, series connection, and other idealized models to predict the modulus of elasticity of hardened concrete, Ec. Conceivably, these assumptions were confined to constant stress/strain modes and idealized aggregate configurations such that theoretical numerical solutions could be made possible. This idealization may in fact have a profound impact on the calculated stress distributions in aggregate and mortar, particularly along the interface areas [5,6].

NUMERICAL ANALYSIS

This study adopted a numerical method to simulate the stress distribution of the mortar and aggregate in concrete. A cylindrical Central Aggregate Model is shown in Fig.1. Using the ANSYS program, the E value can be determined from the stress-strain relation curve. The stresses are assumed to be linear. Based on the results, the relationship among Ea, Em, Ec and Va is obtained.

Verifying the reliability of numerical calculation

The Desize, Lesize and Smartsize mesh methods in the ANSYS program were used to test the reliability of the numerical calculation. A cylindrical concrete model encompassing a single aggregate type (shown in Fig.1) was analyzed using the Solid Element. The specimen size was 50ϕ ×50 mm with a spherical aggregate 40 mm in diameter. The E values were assumed to be Ea = 10 GPa, Em = 40 GPa. The Poisson's ratios (ν) for the aggregate and mortar were both 0.18. After applying an axial load of 10 MPa to the concrete model, the axial deformation and stress were calculated.

The convergence from the above three mesh methods is shown in Fig. 2. In the total node number and concrete axial deformation comparison, it was found that the Desize mesh method produced the least number of nodes, about 500. Axial deformation convergence appeared. In the Lesize mesh method, the number of nodes was about 700 when the axial deformation converged. In the Smartsize mesh method, convergence occurred within 200 nodes. These results indicate that a fewer number of nodes were required for the Smartsize mesh method to reach axial deformation convergence. From the deformation aspect, this is considered the ideal mesh method.

From the relationship between the total number of nodes and the maximum principal aggregate stress (refer to Fig. 2), it is shown that the principal aggregate stress in the Desize method starts to converge when the total number of nodes reaches about 3300. About 2800 nodes are required for the Lesize mode and about 800 nodes for the Smartsize mode. The Smartsize mode is the best mesh method.

From the relationship between total number of nodes and the maximum principal mortar stress (refer to Fig. 2), the Desize mode converges when the total number of nodes reaches about 3300. In the Lesize mode, although there is a variation in the principal mortar stress, the stress becomes stable and starts to converge when the total number of nodes reaches about 2800. In the Smartsize mode, convergence does not begin until the total number of nodes reaches 800. Comparing the convergence analysis results for the Smartsize and the other two modes, the Smartsize mode will converge at about 2300 nodes. The Smartsize method is therefore the most acceptable. The Smartsize method was used in all analysis models for this study.

Comparison of theoretical and numerical solutions of Ec

To make sure the E value from the numerical analysis is reliable, four theoretical models and equations were used for comparison. These models were the Voigt model, Reuss model, Popovics model, and Counto model, as shown in Fig. 3.

The Voigt model assumes that the two materials (aggregate and mortar) are in parallel combination. Under load conditions, the strain behavior of the two materials is equal to the constant strain mode. If the E values (Ea and Em) are known, the Ec of the composite material is calculated from the following equation:

$$Ec = VaEa + VmEm \tag{1}$$

To emphasize the difference between the elastic modulus of these two materials, this research used Ea = 10 GPa, Em = 40 GPa, ν=0.18, various Va from 10 % to 90 % and every 10% increment was a set, so there were nine sets for analysis. The Ec values from all sets were compared with the results from the theoretical equation shown in Fig. 4. It was found that the Ec values from these two methods were nearly the same. This indicates that the numerical model used in the ANSYS program to calculate the Ec value can reach an excellent result under the constant strain mode.

The Reuss model assumes that the two materials are in serial combination. Under load conditions, the stress behavior of the two materials is equal to the constant stress mode. If the E values (Ea and Em) are known, the Ec of the composite material is calculated using the following equation:

$$E_c = \frac{1}{\frac{V_a}{E_a} + \frac{V_m}{E_m}} \tag{2}$$

The comparison of the Ec values from the numerical analysis and theoretical equation is shown in Fig. 4. It indicates that the numerical model in this research has an acceptable result under the constant stress mode.

The Popovics model is the sum of Eq. (1) and (2), the equation can be expressed as:

$$Ec = \frac{1}{2}(Ec,_{voigt}) + \frac{1}{2}(Ec,_{\mathrm{Re}uss}) \tag{3}$$

The comparison result (Fig. 4) shows that the maximum error between the numerical and theoretical solutions is about 3 %. The reason is that the stress and strain behavior are not in the constant stress or constant strain modes. The solutions calculated from the theoretical model will be different from those of numerical analysis.

The composite material model using Counto model assumes a square shaped aggregate inside the center of a hexahedron of mortar. If the E values of the two materials are known, the Ec of the composite material can be calculated using the following theoretical equation:

$$\frac{1}{E_c} = \frac{1-\sqrt{V_a}}{E_m} + \frac{\sqrt{V_a}}{\sqrt{V_a}E_a + \left(1-\sqrt{V_a}\right)E_m} \tag{4}$$

The comparison between numerical analysis results and the theoretical calculation results is shown in Fig. 4. It was found that, for Em/Ea=4 (a big difference between E values of two materials), the error between the numerical solution and the theoretical solution is large, especially when Va increases from 20 % to 30 %. There is nearly a 7 % error between the numerical and theoretical solutions. The reason is that in the derivation process of the theoretical equation, the composite materials in the parallel combination part are assumed to have a constant strain mode and the material in the serial combination part are assumed to have a constant stress mode. In actual Counto model cases, the stress and strain behavior are not in the constant stress or constant strain modes. The binding mechanism involves complicated stress/strain behaviors. This can be proven from the Central Aggregate Model. The real stress distribution result in the Central Aggregate Model (100ϕ ×100 mm with a 60 mm aggregate) is shown in Fig.5. Complicated stress/strain behaviors were involved that did not produce a constant stress/strain mode. As Em/Ea=2, the maximum error is about 2 %. It is concluded that when the E values of the two materials are close, the composite material in the constant stress and constant strain mode assumption becomes more acceptable.

In summary, the result shows that the concrete model established using the numerical method is closer than the theoretical model to the actual concrete stress distribution.

Effect of poisson's ratio on Ec

In real concrete material, the Poisson's ratio ν varies for different aggregate and mortar. The effect on Ec using various aggregate and mortar ν values is shown in Fig. 6. In the numerical simulation assumption, Ea, Em is 10 GPa when Va is 35 %. The ν of the mortar is 0.18 and the ν of the aggregate varies from 0.15 to 0.21 with an increment of 0.01. There are seven sets of data. The result in the figure indicates that Ec is hardly affected by the aggregate and mortar at different ν values. The deviations caused by different ν values are within 0.02%.

Relation among Ea, Em and Ec

To establish the relationship among Ea, Em and Ec, this study used Central Aggregate Model with Va from 10 % to 50 % and an Em / Ea ratio from 0.3 to 6.0. The results are shown in Fig. 7. Changing the size of the concrete model using a fixed aggregate size (10 mm) controls the volume fraction of the aggregate (Va). The aggregate is placed in the center of the model. In the Em / Ea < 1 (the upper half in the figure) case, when Em / Ea is fixed, Ec/Em will increase with increasing Va. When Em / Ea decreases, the difference between Ec and Em (or the ratio) becomes bigger. This is because the aggregate is a reinforcing particle. The Ec value will increase because of the increase in high E material. This is more obvious as Va increases. In the Em / Ea > 1 (the lower half in the figure) case, when Em / Ea is fixed, Ec/Em will decrease with increasing Va. When Em / Ea increases, the difference between Ec and Em becomes bigger. This is a lightweight aggregate concrete case. The aggregate particle is a weak material, and the Ec value will decrease because of the increase in low E material. This is also more obvious as Va increases.

Determination of Ea equation

The numerical results show that Ea, Em and Va are the important factors that affect Ec. Changing the Poisson's ratio of the aggregate and mortar has little effect on Ec. Moreover, based on the theoretical equations recommended in the references [4], Va, Ec, and Em should be used as the main variables. The equation presented with quadratic polynomial is acceptable. Therefore, the complete quadratic polynomial using the 3 variables mentioned above was used to precede the Ea regression equation. The function is as follows:

$$Ea = f(Va,\ Ec,\ Em,\ VaEc,\ VaEm,\ EcEm,\ Va^2,\ Ec^2,\ Em^2) \quad (5)$$

The numerical analysis result shown in Fig.7 is regarded as the reference for selecting the points used in the function. Referring to the mechanical properties of concrete, this study assumed the 5 Ec values as 10, 15, 20, 25, and 30 GPa. The 7 Em/Ea values are 3.0, 2.0, 1.5, 1.0, 0.7, 0.5, and 0.4. The 6 Va values are 0.25, 0.3, 0.35, 0.4, 0.45, and 0.5. A total of 210 sample data sets were acquired. The Shazame program was used to analyze the selected sample points. The regression equation can be determined as

$$Ea = 1251.2Va + 6.67Ec - 5.4Em - 8.8VaEc + 8.3VaEm - 0.59\times10^{-4}EcEm - 4479Va^2 + 0.25\times10^{-4}Ec^2 + 0.33\times10^{-4}Em^2 + 938.6 \quad (6)$$

Statistical quantities related to the regressed relationship are illustrated in Table 1, in which the values of the standard error, T-ratio and P represent the error, significance and the extent of data concentration, respectively. It can be seen that the coefficients associated with the Va and Va^2 terms correspond to extremely large standard errors, less significance and a worse data concentration in the regression. Accordingly, it can then be concluded that these two terms can be deleted without affecting the accuracy.

In further analysis, it was assumed that the Ea equation has only 7 other terms and constants. Putting the sample data into calculation, the formula is determined as follows:

$$Ea = 6.8Ec - 5.5Em - 8.9VaEc + 8.4VaEm - 0.58\times10^{-4}EcEm + 0.24\times10^{-4}Ec^2 + 0.33\times10^{-4}Em^2 + 732.58 \quad (7)$$

Although the R-square is almost equal to 1, the Ea regression equation is still complicated and has too many terms, which is inconvenient for practical use. In order to simplify the terms in the equation, the parameters were re-selected based on the theoretical equation. After several analyses, the equation was determined as follows:

$$Ea = 6.3Ec - 5.1Em - 9.1VaEc + 8.7VaEm \quad (8)$$

Table 2 shows that each term has more accuracy. Comparing with the previous analysis result, the standard errors are greatly decreased, the T-ratio in each term adds up in the equation, and R-square of the equation also reaches 0.9924. This result shows that the equation has high reliability and is a simple formula good for practical use.

EXPERIMENTAL PROCEDURE AND RESULTS

Concrete and mortar specimens were made from normal weight sand, Portland cement and two different aggregates, normal weight and lightweight. The lightweight aggregates used were expanded shale produced in Taiwan with a particle density of 1200 kg/m^3 - 1400 kg/m^3 and expanded clay produced in China with a particle density of 800 kg/m^3 - 1000 kg/m^3. The aggregate sizes were near 20 mm. The water-cement ratio was kept at 0.4 throughout the test program. In order to keep the effective water-cement ratio as constant as possible, the lightweight aggregate was immersed in water for 30 minutes before mixing and then surface dried with a towel. Portland cement, water, and normal weight sand were firstly mixed and then placed one aggregate in the center of mortar to form Central Aggregate Model. Specimens were cured in a curing room (23 °C, R.H. > 95 %) until the time for testing. In order to determine various E values for the mortar and concrete at different compressive strengths, this research measured different Ec and Em values at different specimen curing ages. The E values were measured according to the ASTM Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression (C-469). A test machine with a 2000 KN load capacity was used. The E values for normal weight aggregate could be measured directly on the cylinder cores or calculated from the Ea equation. The results were compared to determine the validity of the proposed model and Ea equation. Having achieved a close agreement between the calculated and tested data, a series of lightweight aggregate concrete and mortar cylinders were subsequently tested to obtain the Ec, Em and Va values. The cylinder sizes were selected as

- normal concrete and mortar:150ϕ × 150 mm (spherical normal aggregate size is near 120 mm)
- normal weight aggregate core: 44ϕ × 44 mm (cored from concrete specimens)

- lightweight aggregate concrete and mortar: 28 $\phi \times 28$ mm (lightweight aggregate size is near 20 mm)

The total number of mortar and concrete specimens was 27, with 3 cores. The mortar was blended first and then mixed with a single aggregate to make the Central Aggregate Model specimen. In addition to the concrete specimen, a mortar specimen was also made using the same batch for comparison.

The mortar and concrete compression test results were used to determine Ec and Em respectively. The Ea values are shown in Tables 3 and 4. From the Ea value comparison determined using the Ea equation and core cylinders from the normal weight aggregate (shown in Table 3), the errors were within 10 %. The result means that the Ea equation has high reliability. The Ea results for the lightweight aggregate based on the Ea equation are shown in Table 4. It was found that the Ea of expanded shale from the six sets were from 8.0 GPa to 11.7 GPa. The average was 10.25 GPa and the variation stayed within 22 %. The Ea of expanded clay were from 5.5 GPa to 7.8 GPa. The average was 6.8 GPa and the variation stayed within 20 %. This indicates that there is consistency in determining Ea using the proposed equation. This means that using the Central Aggregate Model to simulate a single aggregate in concrete and deriving the Ea equation is reasonable and acceptable.

ACKNOWLEDGMENT

The authors would like to thank Dr. T. D. Lin for his helpful suggestions and H. C. Chan for her numerical analysis assistance during this project.

CONCLUSION

This study proposed the Central Aggregate Model for simulating the aggregate inside concrete and establishing the E relationship among the aggregate, mortar and concrete. With simple test verification, the following results were obtained:

(1) By comparing the numerical analysis result with the theoretical solution, it was proven that using the ANSYS program to analyze the Central Aggregate Model is acceptable and the concrete model established using the numerical method is closer to the actual stress distribution in concrete.

(2) In the Counto model, when the E ratio of the two materials increases, the Ec error will increase. This is because the theoretical model assumed constant stress and constant strain modes, which did not match the actual stress condition in the concrete.

(3) This study indicates the practicability of using the numerical method to establish the relationship among Ea, Em, Ec and Va based on the Central Aggregate Model. Substituting the experimental results into the Ea equation to determine the E value for lightweight aggregate produced in Taiwan and China was proven to be an excellent and reliable method.

(4) According to the numerical results, an Ea equation for the Central Aggregate Model was evaluated as

$$Ea = 6.3\ Ec - 5.1Em - 9.1VaEc + 8.7VaEm$$

REFERENCES

[1] J. Muller-Rochholz, 1979, "Determination of the Elastic Properties of Lightweight Aggregate by Ultrasonic Pulse Velocity Measurement," The International Journal of Lightweight Concrete. Vol. 1, No.2, pp. 87-90.

[2] A. Ulrik Nilsen, Paulo J.M. Monteiro and Odd E. Gjorv., 1995, "Estimation of the Elastic Moduli of Lightweight Aggregate," Cement and Concrete Research, Vol.25, No. 2, pp. 276-280.

[3] Yang, C. C., 1997, "Approximate elastic moduli of lightweight aggregate," *Ibid*, Vol. 27, No. 7, pp. 1021-1030.

[4] Sidney Mindess, J. Francis Young, 1981, “Concrete,” Prentice-Hall, pp. 339-344.

[5] Hirsch, T. J., 1962, “Modulus of Elasticity of Concrete Affected by Elastic Moduli of Cement Paste Matrix and aggregate,” ACI Journal, Proceedings Vol. 59, Feb. pp. 427-451.

[6] Popovics, S., and Erdey, M., 1970, “Estimation of the Modulus of Elasticity of Concrete-Like Composite Materials,” Materials and Structures, Research and Testing (RILEM, Paris), Vol. 3, No. 16, pp. 253-260.

List of Notations

E_a = elastic modulus of aggregate

E_c = elastic modulus of concrete

E_m = elastic modulus of mortar

V_a = volume fraction of aggregate

V_m = volume fraction of mortar

ν = poisson's ratio

List of Table Captions

Table 1 Results of regression

Table 2 Results of regression

Table 3 Comparison of experimental and Ea equation results

Table 4 Determination of Ea based on Ea equation

List of Figure Captions

Figure 1 Central Aggregate Model

Figure 2 Convergence of axial deformation and principal stress

Figure 3 Theoretical concrete model

Figure 4 Theoretical and numerical solution comparison

Figure 5 Principal stress distribution in the Central Aggregate Model

Figure 6 The Poisson's ratio effect on Ec

Figure 7 Relation among Ea , Em and Ec

Table 1 Results of regression

Ea = f (Va, Ec, Em, VaEc, VaEm, EcEm, Va^2, Ec^2, Em^2)

Parameter	Coefficient	Standard error	T-Ratio	P-value
Va	1.25×10^{3}	2.46×10^{4}	5.09×10^{-2}	0.959
Ec	6.68×10^{0}	2.74×10^{-1}	2.44×10^{1}	0.000
Em	-5.40×10^{0}	2.57×10^{-1}	-2.10×10^{1}	0.000
VaEc	-8.79×10^{0}	3.28×10^{-1}	-2.68×10^{1}	0.000
VaEm	8.34×10^{0}	3.01×10^{-1}	2.77×10^{1}	0.000
EcEm	-5.90×10^{-5}	7.08×10^{-6}	-8.33×10^{0}	0.000
Va^2	-4.48×10^{3}	2.97×10^{4}	-1.51×10^{-1}	0.880
Ec^2	2.49×10^{-5}	5.21×10^{-6}	4.78×10^{0}	0.000
Em^2	3.26×10^{-5}	2.97×10^{-6}	1.09×10^{1}	0.000
constant	9.39×10^{2}	4.90×10^{3}	1.92×10^{-1}	0.848

R-square=0.9956

Table 2 Results of regression

Ea = f (Ec, Em, VaEc, VaEm)

Parameter	coefficient	Standard error	T-Ratio	P-value
Ec	6.332	0.1391	45.66	0.000
Em	-5.122	0.1351	-37.91	0.000
VaEc	-9.103	0.3123	-29.15	0.000
VaEm	8.735	0.2990	29.28	0.000

R-square=0.9924

Table 3 Comparison of experimental and Ea equation results

Normal Weight Aggregate Concrete	Curing time	Em (Gpa)	Ec (Gpa)	Va	Experimental Ea (Gpa)	Equation Ea (GPa)	Error (%)
	30 hrs	$15.1\begin{pmatrix}14.9\\14.8\\15.5\end{pmatrix}$	19.3	0.34	32.3	29.5	-8.6
			18.2	0.30	29.8	27.4	-8.1
			19.0	0.30	30.5	30.2	-1.0

Table 4 Determination of Ea based on Ea equation

Light Weight Aggregate	Curing time	Em (GPa)	Ec (GPa)	Va	Ea (GPa)	$\frac{Ea - Ave.}{Ave.} \times 100$ (%)
Expanded Shale (Taiwan)	24 hrs	$12.9\begin{pmatrix}11.9\\12.9\\13.9\end{pmatrix}$	11.8	0.28	$9.8\begin{pmatrix}9.9\\8.0\\11.4\end{pmatrix}$	-3.4
			11.3	0.28		-22.0
			12.2	0.24		+11.2
	42 hrs	$18.2\begin{pmatrix}18.4\\18.1\\18.2\end{pmatrix}$	15.4	0.28	$10.7\begin{pmatrix}9.3\\11.2\\11.7\end{pmatrix}$	-9.3
			16.0	0.25		+9.3
			16.1	0.26		+14.1
Expanded Clay (China)	20 hrs	$11.4\begin{pmatrix}10.6\\11.4\\12.1\end{pmatrix}$	9.6	0.27	$6.8\begin{pmatrix}5.5\\7.1\\7.8\end{pmatrix}$	-19.1
			10.0	0.27		+4.4
			10.2	0.27		+14.7

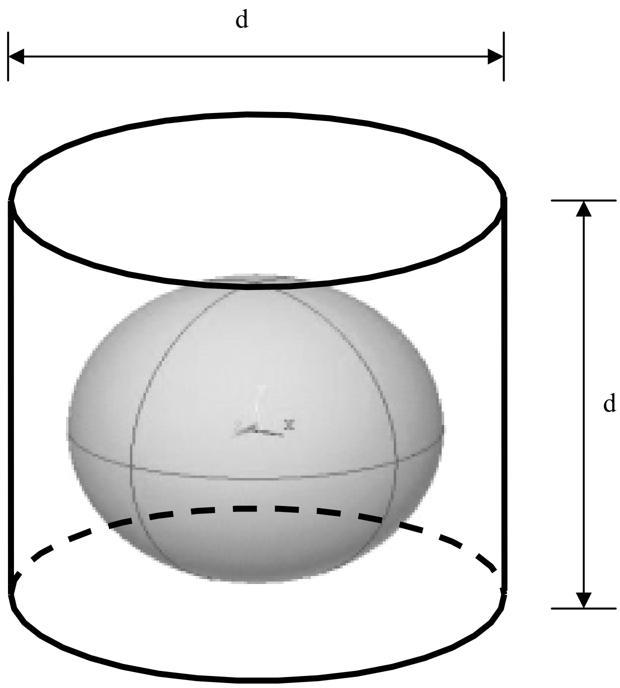

Figure 1 Central Aggregate Model

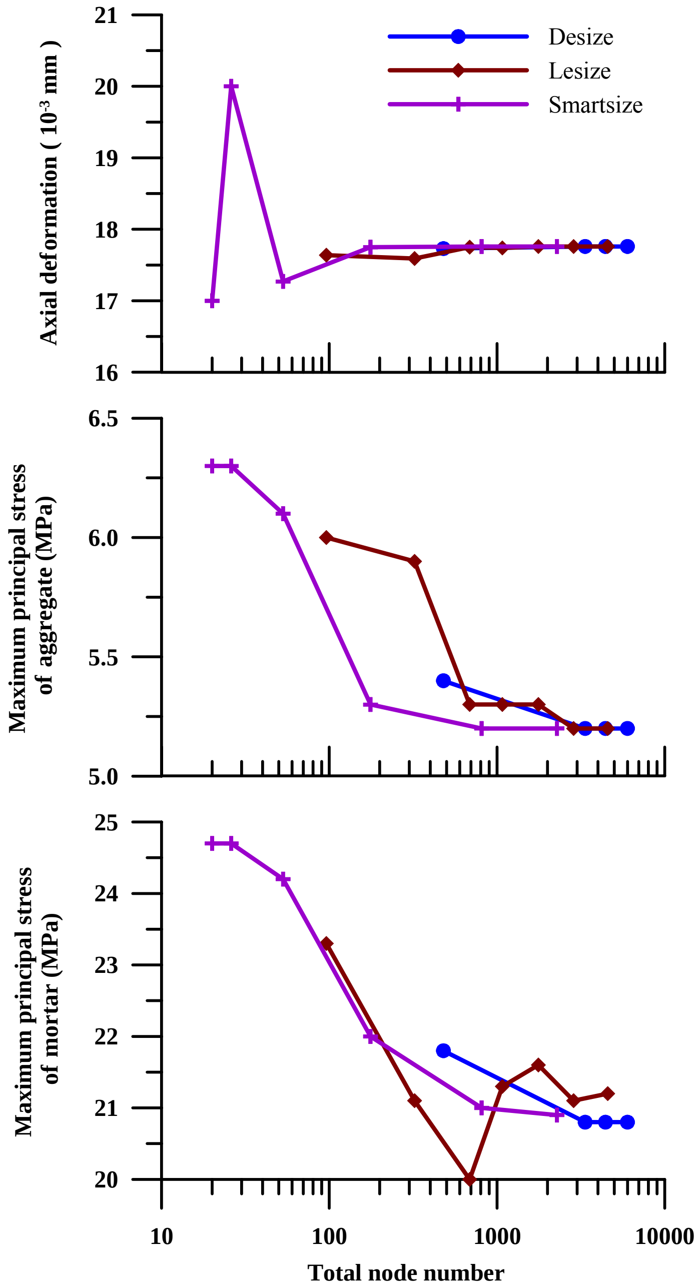

Figure 2 Convergence of axial deformation and principal stress

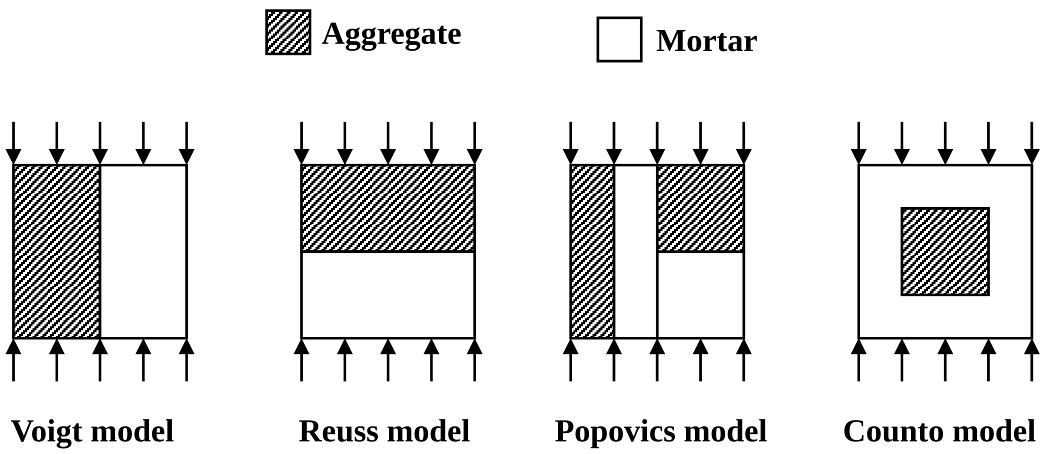

Figure 3 Theoretical concrete model

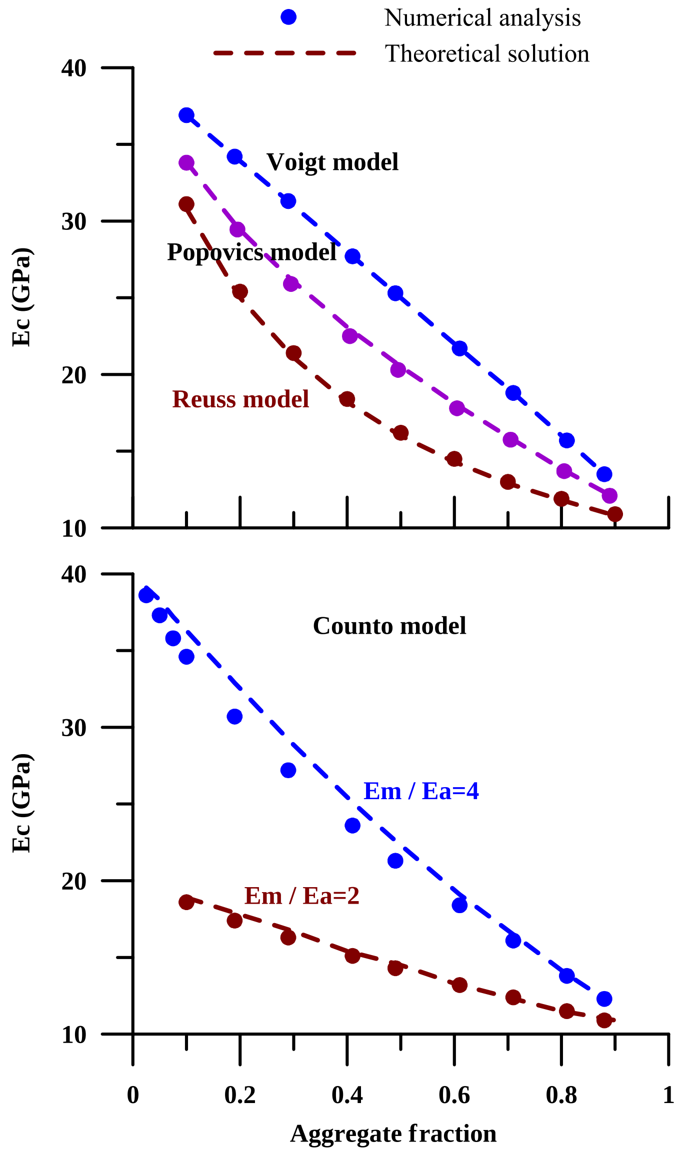

Figure 4 Theoretical and numerical solution comparison

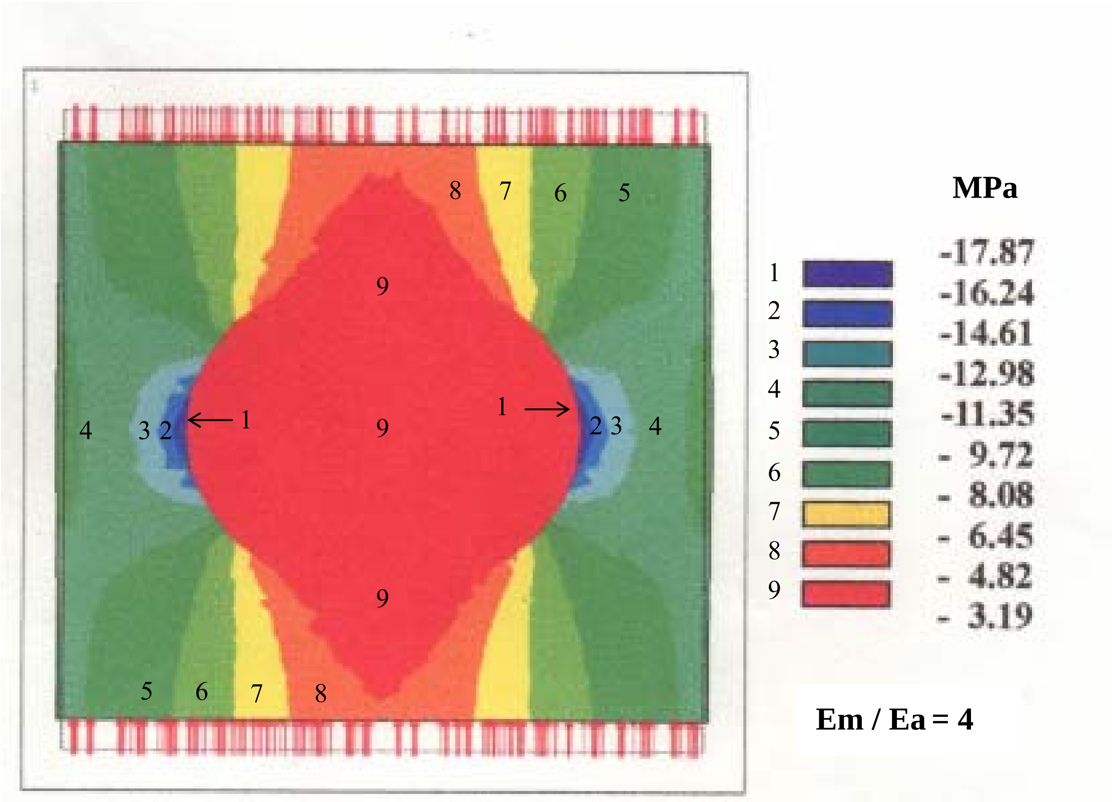

Figure 5 Principal stress distribution in the Central Aggregate Model

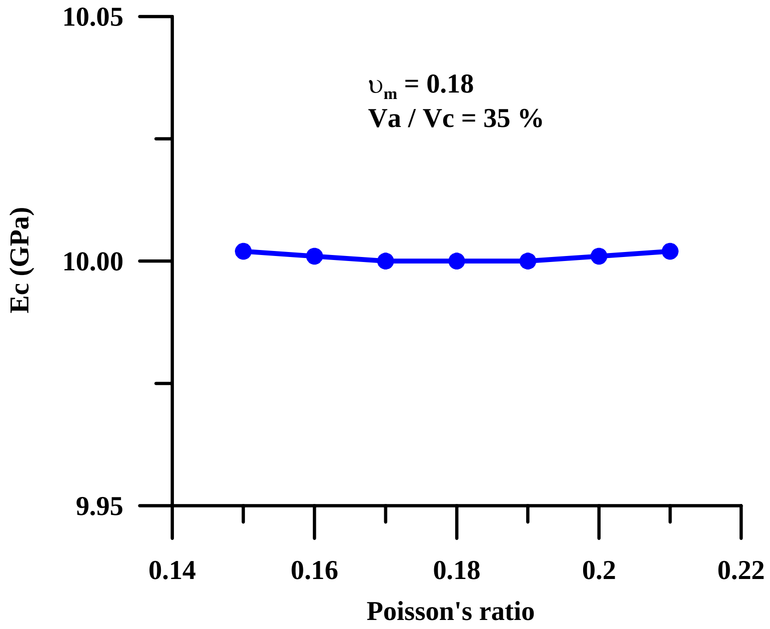

Figure 6 The Poisson's ratio effect on Ec

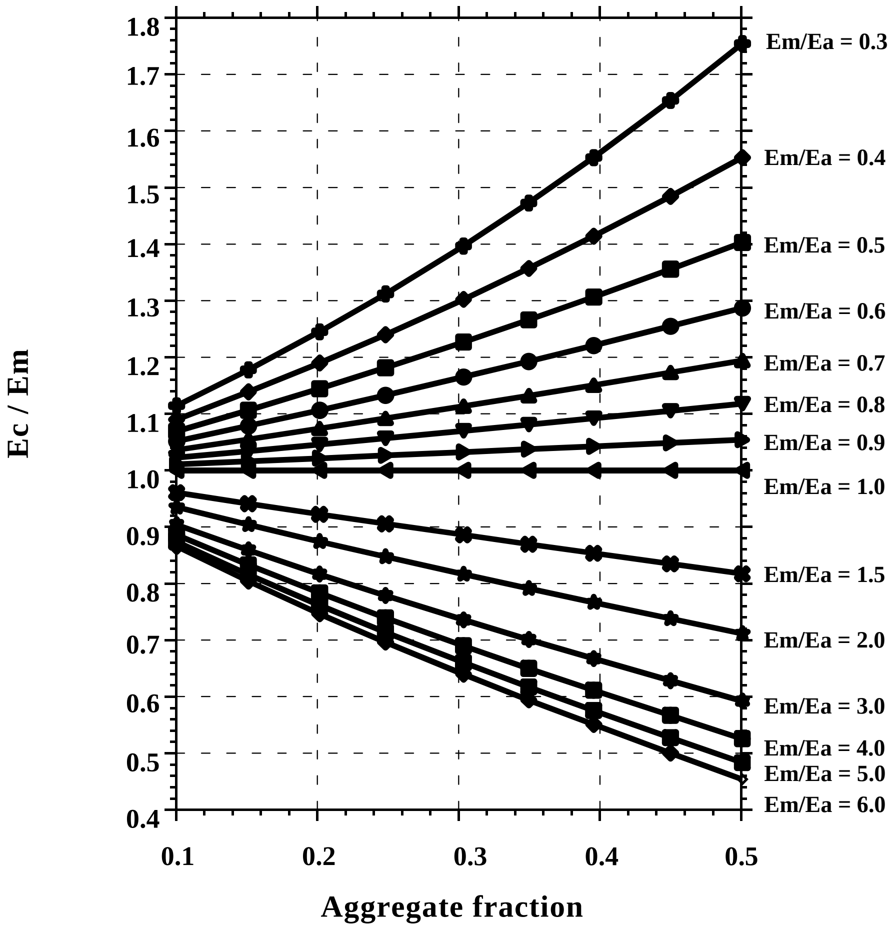

Figure 7 Relation among Ea, Em, and Ec