

國立中興大學 112 學年度碩士班招生考試試題

科目：工程數學

系所：土木工程學系 甲組

本科目試題共 1 頁

1. Find a general solution of $4x^2y'''' + y'' = 2$. (10%)
2. Solve the boundary value problem $\frac{d^2}{dx^2}\left(c \frac{d^2y}{dx^2}\right) = p_o \frac{x}{L}$ with $y(0) = y'(0) = y''(L) = y'''(L) = 0$, in which c , p_o , and L are constants. (10%)
3. Solve the initial value problem $y'(t) = -\int_0^t y(t-\tau)e^{-2\tau} d\tau$ with $y(0) = 1$. (10%)
4. Consider the system of ODEs $\mathbf{y}' = \mathbf{A}\mathbf{y}$ in which \mathbf{y} is a three-by-one column vector and
$$\mathbf{A} = \begin{bmatrix} 2 & -6 & 0 \\ 2 & 0 & 4 \\ 0 & 3 & 2 \end{bmatrix}.$$
 - (a) Find eigenvalues and corresponding eigenvectors for \mathbf{A} . (10%)
 - (b) Find a general solution of the system. (10%)
5. Solve the partial differential equation
$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2}$$
which satisfies $u(x=0, t) = 0$, $\frac{\partial u}{\partial x}(x=L, t) = 0$, $u(x, t=0) = 0$ and $\frac{\partial u}{\partial t}(x, t=0) = v_o \sin \frac{3\pi x}{2L}$. Where c , L , and v_o are constants. (30%)
6. Consider a scalar function $f(x, y, z) = 2x + 3yz - 4xyz$, a point $P: (1, 2, 1)$ and a vector $\mathbf{a} = [2, -1, 2]$.
 - (a) Determine the directional derivative of f at P in the direction of \mathbf{a} . (5%)
 - (b) Determine the direction in which f has the maximum rate of change at P . (5%)
7. Determine the length of a curve $C: x^2 + y^2 = 16, x = 4 \sin(z/4)$ for $0 \leq z \leq 12$. (10%)