

國立中興大學

111 學年度

碩士班考試入學招生

試題

招生系所：土木工程學系甲組

科目名稱：工程數學

國立中興大學 111 學年度碩士班招生考試試題

科目：工程數學

系所：土木工程學系 甲組

本科目可以使用計算機

本科目試題共 1 頁

1. Find a general solution for $x^2 y' + (1 - y)y = -6$. (10%)
2. Find a general solution for $2y' - 2y = xy^{-1}(1 + 2 \ln x)e^{2x}$ (10%)
3. Solve the initial value problem $y'' + \frac{\pi^2}{64}y = \delta(t - 2) \cos \frac{\pi}{8}t$ with $y(0) = y'(0) = 0$. δ is the Dirac delta function. (10%)

4. Consider a system of ODEs $\mathbf{y}'' + \mathbf{A}\mathbf{y} = \mathbf{f}$, in which

$$\mathbf{A} = \begin{bmatrix} 3 & -3 \\ -2 & 4 \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} 10 + 3 \sin t \\ -10 + 2 \sin t \end{bmatrix}, \text{ and}$$

\mathbf{y} is a two by one column vector satisfying the initial conditions $\mathbf{y}(0) = \mathbf{y}'(0) = \mathbf{0}$.

- (a) Find eigenvalues and corresponding eigenvectors for \mathbf{A} . (10%)
- (b) Solve the initial value problem. (10%)

5. Solve the partial differential equation

$$\frac{\partial^2 u(x, z)}{\partial x^2} + \frac{\partial^2 u(x, z)}{\partial z^2} = 0$$

in which $u(x, z)$ satisfies

$$u(\pm a, z) = u(x, -b) = 0, \quad \frac{\partial}{\partial z} u(x, b) = w_0.$$

w_0, a and b are positive constants.

6. Solve the partial differential equation

$$\frac{\partial u(x, t)}{\partial t} - c \frac{\partial u(x, t)}{\partial x} = 0$$

with $u(x, 0) = u_0(x)$. c is a positive constant.

7. Evaluate

$$I = \oint_C \left(\frac{y}{x} + yz \right) dx + (\ln x + xz) dy + xy dz$$

in which C is a closed path along the intersection of a plane $x + 3y - 6z = 0$ and a cylinder $x^2 + y^2 = 4$.

(10%)