

國立中興大學

110 學年度

碩士班考試入學招生

試 題

學系：土木工程學系 甲組

科目名稱：工程數學

國立中興大學110學年度碩士班招生考試試題

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本科目可以使用計算機

本科目試題共 1 頁

- 1 Find a general solution for $2xy' + y^2 = 1$. (10%)
- 2 Find a general solution for $e^{3y}y' = 2(x - e^{3y}) + 1$. (10%)
- 3 Solve the initial value problem $y'' + \omega^2 y = \delta(t - \tau)$ with $y(0) = K_0$ and $y'(0) = K_1$, in which δ is the Dirac delta function, ω , τ , K_0 and K_1 are some constants, $\omega > 0$ and $\tau \geq 0$. (15%)
- 4 Consider a symmetric matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

- (a) Find the eigenvalues and eigenvectors of \mathbf{A} . (15%)
- (b) Find a diagonal matrix \mathbf{D} and a matrix \mathbf{P} that satisfy $\mathbf{D} = \mathbf{P}^T \mathbf{A}^4 \mathbf{P}$. (5%)

- 5 Solve a wave equation

$$\frac{\partial^2 u(x, t)}{\partial t^2} - c^2 \frac{\partial^2 u(x, t)}{\partial x^2} = 0$$

that satisfy boundary conditions $u(0, t) = 0$ and $u_x(L, t) = 0$, and initial conditions $u(x, 0) = 0$ and $u_t(x, 0) = v_0$, where $u_x = \partial u / \partial x$ and $u_t = \partial u / \partial t$. c , L and v_0 are constants. $c > 0$ and $L > 0$. (25%)

- 6 Show the set of function, $2x$, $x + 2$ and x^2 , defined on $x \in \mathbb{R}$ are linearly independent. (10%)

- 7 Consider a set of functions which have the form

$$\phi_n(x) = \sqrt{\frac{2n+1}{2}} \sum_{m=0}^M (-1)^m \frac{(2n-2m)!}{2^n m! (n-m)! (n-2m)!} x^{n-2m}$$

where $n = 0, 1, 2, \dots$ represent any positive integers, and $M = n/2$ or $(n-1)/2$, whichever is an integer, e.g. $M = 2$ when $n = 4$ or 5 . These functions are orthogonal on the interval $-1 \leq x \leq 1$,

$$\int_{-1}^1 \phi_m(x) \phi_n(x) dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$$

If we try to express $5x^4 + x^3$ as a series of these functions on the interval $-1 \leq x \leq 1$,

$$5x^4 + x^3 = \sum_{n=0}^{\infty} c_n \phi_n(x),$$

please determine the values of c_1 and c_6 . (10%)