

國立中興大學

109 學年度

碩士班考試入學招生

試 題

學系：土木工程學系甲組

科目名稱：工程數學

109學年度碩士班招生考試試題

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系所： 土木工程學系 甲組

本科目可以使用計算機

本科目試題共 **1** 頁

- 1 Find a general solution for $e^{3y}y' = 2(x + e^{3y}) + 3$. (10%)
- 2 Solve the initial value problem $y'' + 2y' + 4y = \delta(t - 1)$ with $y(0) = y'(0) = 0$. δ is the Dirac delta function. (10%)

- 3 Find all eigenvalues and corresponding eigenvectors for $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. (10%)

- 4 If $\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$ is a n by n square matrix and $\det \mathbf{A} = \alpha$, find

(a) $\det(\mathbf{A}\mathbf{A}^T)$, (5%)

(b) $\det(c\mathbf{A})$ in which c is a scalar. (5%)

- 5 Consider a partial differential equation

$$\frac{\partial^2 u(x, t)}{\partial t^2} + \alpha^2 \frac{\partial^4 u(x, t)}{\partial x^4} = 0$$

in which $u(x, t)$ is a periodic function of t satisfying

$$u(0, t) = u(L, t) = \frac{\partial^2 u(x, t)}{\partial x^2} \Big|_{x=0} = \frac{\partial^2 u(x, t)}{\partial x^2} \Big|_{x=L} = \frac{\partial u(x, t)}{\partial t} \Big|_{t=0} = 0$$

and

$$u(x, 0) = v_1 \sin\left(\frac{\pi x}{L}\right) + v_2 \sin\left(\frac{2\pi x}{L}\right).$$

α , v_1 , v_2 and L are some positive constants.

(a) Solve the PDE. (25%)

(b) What is the period of $u(x, t)$? (5%)

- 6 Solve the partial differential equation

$$\frac{\partial u(x, t)}{\partial t} + c \frac{\partial u(x, t)}{\partial x} = 0$$

with $u(x, 0) = u_0(x)$. c is a constant. (10%)

- 7 Consider a scalar function $f(x, y, z) = xz - yz$ and a vector $\mathbf{a} = [1, 2, 3]$.

(a) Calculate the directional derivative of f at point $P: (1, 0, 3)$ in the direction \mathbf{a} . (5%)

(b) Find the direction in which f has maximum rate of change at P . (5%)

- 8 Evaluate

$$I = \iint_S xz^2 dydz + x^2 y dzdx - xy^2 dx dy$$

in which S is a closed surface consisting of the cylinder $x^2 + y^2 = 4$ ($0 \leq z \leq 3$) and the circular disks $z = 0$ and $z = 3$ ($x^2 + y^2 \leq 4$). (10%)