

本科目可以使用計算機

本科目試題共 1 頁

1. Solve the following nonhomogeneous linear system of ordinary differential equations with the given initial values,

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} e^{2t} - 4t \\ t + 2 \end{bmatrix}, \quad y_1(0) = -1, \quad y_2(0) = -2$$

and show the solutions of  $y_1(x)$  and  $y_2(x)$ . (20%)

2. Solve the following initial value problem of a linear ordinary differential equation.

$$y''(t) + 4y(t) = \begin{cases} 4\cos t & , 0 \leq t \leq \pi \\ 0 & , t > \pi \end{cases}, \quad y(0) = \frac{8}{3}, \quad y'(0) = 1 \quad (20\%)$$

3. Calculate the Fourier series of the following periodic function,  $h(x)$ .

$$h(x) = 3x^2, \quad -\pi < x < \pi$$

and  $h(x + 2\pi) = h(x)$  (20%)

4.  $(\frac{x}{2} + 2x^3)$ ,  $(2x^3 + \frac{1}{x})$ , and  $(\frac{4}{x} + 2x^3 + \frac{x}{8})$  are three solutions of the following linear ordinary differential equation among others

$$x^2 y'' + a(x)y' + b(x)y = g(x)$$

Find  $a(x)$ ,  $b(x)$ , and  $g(x)$ . (20%)

5. Solve the following partial differential equation to find the solution of  $u(x, t)$

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq l, \quad t \geq 0$$

which has the boundary conditions:  $u(0, t) = u_x(l, t) = 0$ ;

and the initial condition:  $u(x, 0) = f(x)$ ,  $u_t(x, 0) = 0$ . (20%)