

本科目可以使用計算機

本科目試題共 1 頁

1. Solve the initial value problem and plot the time history of  $y$  for the following ordinary differential equation: (10%)

$$y'' + 0.4y' + 9.04y = 0 \text{ with } y(0) = 0, y'(0) = 3.$$

2. Find (10%)

$$(a) L[2\cosh(\omega t) + 3\sinh(\omega t)], \quad (b) L^{-1}\left(\frac{2s}{s^2 + 25s + 150}\right).$$

3. Compute the (a) divergence, (b) curl, and (c) gradient for  $3y^3\vec{i} - 2xz^2\vec{j} + xyz\vec{k}$ . (20%)

4. Give the following matrix:  $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$  (20%)

- (a) Find the eigenvalues and eigenvectors of  $A$ .  
(b) Diagonalize  $A$  and then use the results to find  $A^{10}$ .

5. Interpret the physical meanings of Fourier series and Fourier transform, respectively. (10%)

6. Solve the following partial differential equation (20%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - G \quad (G: \text{acceleration of gravity})$$

with boundary conditions:  $u(0, t) = 0$

$$u(L, t) = 0$$

$$\text{and initial conditions: } u(x, 0) = f(x) = \begin{cases} \frac{2x}{L}, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2}{L}(L-x), & \text{if } \frac{L}{2} < x < L \end{cases}$$

$$u_t(x, 0) = 0$$

7. Evaluate  $\int_{-\infty}^{\infty} \frac{\sin mx}{x(x^2 + k^2)} dx \quad (m \geq 0, k > 0)$  (10%)