

本科目可以使用計算機

本科目試題共 1 頁

1. $(\frac{-x^2}{3} + 2x^{-1})$, $(\frac{-x^2}{3})$, and $(\frac{-x^2}{3} - x^3)$ are three solutions of the following linear ordinary differential equation among others

$$x^2 y'' + a(x)y' + b(x)y = g(x)$$

Find (1) $a(x)$, $b(x)$, and $g(x)$; (15%)

(2) the general solution of the ODE, $y(x)$. (5%)

2. (1) Considering a matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$, find its eigenvalues and eigenvectors. (8%)

(2) Use the results of (1) to solve the following system of ordinary differential equations

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 10 \cos t \\ -10 \sin t \end{bmatrix}$$

Find the general solutions of $y_1(x)$ and $y_2(x)$. (12%)

3. Calculate the general solution of the following initial value problem of a linear ordinary differential equation.

$$y''(t) + 3y'(t) + 2y(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & t > 1 \end{cases}, \quad y(0) = 0, \quad y'(0) = 0 \quad (20\%)$$

4. Calculate the Fourier sine series of the following periodic function, $f(x)$.

$$f(x) = \begin{cases} x, & 0 \leq x \leq 3 \\ x-6, & 3 < x < 6 \end{cases}$$

and $f(x+6) = f(x)$ (20%)

5. Solve the following partial differential equation to find the solution of $u(x, t)$

$$\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq l, \quad t \geq 0$$

which has the boundary conditions: $u(0, t) = u(l, t) = 0$;

and the initial condition: $u(x, 0) = x$. (20%)