國立中興大學104學年度碩士班招生考試試題

科目：工程數學　　系所：土木工程學系乙組

本科目可以使用計算機　　本科目試題共 / 頁

1.Solve the following ODEs　（15%）

(1) $x^2y''+2xy'-2y=0,\ y(1)=-1,\ y'(1)=-7.$

(2) $(D^2-1)D^3(D^2+D+2)y=0,\ D=\dfrac{d}{dt}.$

2. Find the Laplace / inverse Laplace transforms　（10%）

(1) $L\left\{\sinh^2 t - t^{1/2}\right\}$.　(2) $L^{-1}\left\{\left(\dfrac{s-2}{s-3}\right)e^{-\pi s}\right\}$.

3. Consider the vector differential calculus, if

$f=xz-yz$, and $\vec{A}=y^2\vec{i}+(y^2-x^2)\vec{j}+2Z^2\vec{k}$,

then to find :　（20%）

(1) $\nabla^2(xz\ f)$.　(2) $div\ (grad\ f)$.

(3) $curl\ \vec{A}$.　(4) $grad\ (div\ \vec{A})$.

4. Find the line integral $I=\int_c 2xydx+(x^2-3y^2)dy$,

where curve C is shown in Fig.1.　（10%）

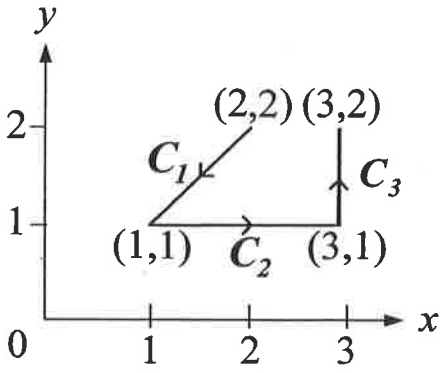

Fig.1.

5.Expand $f(x)=x,\ -2<x<2$, in a Fourier series.　(6%)

6.(1) Solve the following partial differential equation　(15%)

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}-G$$　(G: acceleration of gravity)

with two boundary conditions: $u(0, t) = 0$ and $u(L, t) = 0$,

and two initial conditions: $u(x, 0) = f(x)$ and $u_t(x, 0) = 0$.

(2) Also illustrate the physical meanings of eigen values and eigen functions of this equation.　(10%)

7.Evaluate

(1) $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{(x-1)(x^2+3)}$.　(8 %)

(2) $\displaystyle\oint_\Gamma\frac{e^{z^3}}{(z-i)^3}dz$,　(6 %)

where Γ is a closed path that does not pass through i.