Numerical investigations of wind pressure profiles at isolated escarpment, ridge, and hill

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ABSTRACT
A series of numerical simulations were conducted to analyze the turbulent flow fields in lands, typically under a ground condition of Exposure C with isolated escarpments, ridges, and hills, respectively. By varying the extent of the slope in terms of a height-to-length ratio, the calculated velocity profiles at various cross sections were examined and compared to those suggested by the wind design code. Results show that separation generally occurs and leads to flow unsteadiness at leeward of ridges and hills. Within the re-circulating zone, the velocity fluctuation varies in a manner close to a normal distribution. Finally, comparisons were made in terms of the displacement thickness and the applicability of the code suggestions was assessed. It was found that the related codes may require examinations for further possible modifications.

1. Introduction
Unlike in flat territories, the wind flow in mountain areas is usually affected by local topographical features. Depending on the variation of the ground surface, the flow can accelerate, decelerate or even change its direction. To evaluate the wind load on a building in areas with significant changes of ground surface, in ASCE/SEI 7-10 (ASCE 2013), for instance, a correction factor is introduced to account for the topographical effect in cases of isolated two-dimensional escarpments, ridges, and three-dimensional hills. According to the quasi-steady theory, the adoption of this factor allows for a handy way to obtain the wind velocity profiles and further to estimate the building wind loads in the wind direction at the preliminary design stage. Physically, as the wind flows over a ridge or a hill, separation usually occurs and results in leeward flow unsteadiness. It is then important to examine the applicability of the code regarding the description of the wind pressure profiles in territories with such curved ground surfaces.

In the quasi-steady theory, it is assumed that when an air particle with a certain approaching speed impinges on a structure surface, it becomes still. As a result, the dynamic pressure carried by the particle is completely converted into stagnation pressure. Accordingly, to evaluate the wind loads of a structure in the wind direction, this approach leads to a simple way to determine the pressure distribution corresponding to the local wind velocity profile, and the resulting wind load is the outcome of the integration of pressure with respect to the projected area of the structure.

In an open terrain, the ASCE/SEI 7-10 code (ASCE 2013) suggests that the velocity pressure profile to be obtained as

\[ q_z = \frac{L}{2} K_z K_{zt} K_d V^2 I, \]  

where \( q_z \) is velocity pressure; \( \rho \) is density of air; \( K_z \) is wind directionality factor; \( V \) is basic wind speed, and \( I \) is importance factor. The exposure coefficient (\( K_z \)) describes the variation of the velocity profile with height based on power law. Moreover, the topographic factor, \( K_{zt} = (1 + K_1 K_2 K_3)^2 \), is included to reflect the wind speed-up effect.

In areas with isolated 2-D ridges, 2-D escarpments, and 3-D axi-symmetrical hills (see Figure 1), the suggested values of the associated multipliers (\( K_1 \) to \( K_3 \)) can be readily found in the ASCE/SEI 7-10 (ASCE 2013) code (see Figure 2), depending on the type of land exposure and the slope ratio (\( H/L_a \)). It is noted that since the flow results are provided for the analysis at the preliminary design stage, the existence of trees and any other objects above the ground is ignored and the slope surfaces are considered smooth.

2. Related studies
Due to the topographical complexity in mountains, the wind flow pattern is rather different from that in flat territory. Typical wind tunnel experiments on the flow around hills were carried out in two-dimensional (Ferreira et al. 1995; Kim et al. 1997; Carpenter and Locke 1999; Shiau and Hsu 2003) and three-dimensional (Ishihara, Hibi, and Oikawa 1999; Takahashi et al. 2005) cases. Particularly, Lubitz and White (2007) studied the effect of wind direction on the local flow by both field and wind
concentrated on the discussion of the surface roughness effect on the local wind flow in mountain areas. It was found that the roughness could affect not only the occurrence of separation at the slope surface but the extent of flow acceleration/deceleration. On the other hand, a number of numerical flow predictions were successfully performed in complex terrains. Typically, by the adoption of a $k-\varepsilon$ turbulence model, Eidsvik and Utnes (1997) examined numerically the features of hill flows with separation, attachment, and hydraulic transitions. Similar numerical studies were conducted by Kobayashi, Pereira, and Siqueira (1994), Craig et al. (1999), Uchida and Ohya (1999), Kim, Patel, and Lee (2000), and Lun et al. (2003, 2007). Taylor (1998) reviewed aspects of turbulent boundary-layer flow over low hills and other topography and presented the related basic ideas related to modeling. Weng, Taylor, and Walmsley (2000) further proposed additional formulations based on their numerical results flow over hills.

3. Description of the study

A series of numerical computations were carried out and the results were compared to those suggested by the code in areas with isolated escarpments, ridges, and hills. The comparisons include not only the cross-sectional mean velocity profiles but also the effect due to the fluctuating characteristics of flow unsteadiness. In the study, the slope surface was considered smooth and a cosine curve was used to describe the ground shape. Territory C, corresponding to a rural type of ground condition, was considered and the slope ratios ($H/L_h$) varied from 0.1 to 0.6. The corresponding boundary layer thickness ($\delta$) of the approaching flow was taken as 300 m and the power-law $\alpha$ value was equal to 0.15. The height of the escarpment, ridge, and hill ($H$) was chosen as 100 m. The corresponding Reynolds number ($Re=U_\delta H/\nu$; $U_\delta$ is the edge velocity of the approaching flow) is $10^7$.

4. Numerical simulation

4.1. Numerical method

A weakly-compressible-flow method (Song and Yuan 1988) is adopted in the flow calculations. The original form of the method is applicable to inviscid flow cases and is further extended to viscous turbulent flow simulations. Under a barotropic assumption (density is a function of pressures only), for low Mach-number flows the continuity and momentum equations in index forms can be approximated as (Song and Yuan 1988)

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j} (k \mu_j) = 0,$$

(2)
\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j},
\]
(3)

where \( K (= \rho c^2) \) is the bulk modulus of elasticity; \( \rho, p \) and \( c \) are, respectively, density, pressure, and sound speed.

For turbulent flows, a large eddy simulation approach is applied. Accordingly, any quantity, \( g \), can be decomposed into resolvable-scale and subgrid-scale components as
\[
g = \bar{g} + g'.
\]
(4)

Through a space-averaging process, the governing equations become
\[
\frac{\partial \bar{p}}{\partial t} + \frac{\partial \bar{p} \bar{u}_i}{\partial x_j} (K \bar{u}_i) = 0,
\]
(5)

where
\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial (\bar{p}^*/\rho)}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ -\bar{u}_i' \bar{u}_j' - \bar{u}_i \bar{u}_j' - \left( \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \right) \right] + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j}.
\]
(6)

By adopting Reynolds’ averaging assumption as
\[
\bar{u}_i' \bar{u}_j' - \bar{u}_i \bar{u}_j' + \left( \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \right) = 0,
\]
(7)

one obtains
\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial (\bar{p}^*/\rho)}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ -\left( \bar{u}_i' \bar{u}_j' - \frac{1}{3} \bar{u}_i' \delta_j \right) \right] + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j},
\]
(8)

where \( \delta_j \) is the Kronecker delta function and \( \bar{p}^* = \bar{p} + (\rho/3) \bar{u}_i \bar{u}_i \).

As the subgrid-scale stress terms are modeled by
\[
-\left( \bar{u}_i' \bar{u}_j' - \frac{1}{3} \bar{u}_i' \delta_j \right) = \nu S_{ij},
\]
(9)

where
\[
S_{ij} = \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right),
\]
(10)

Equation (8) becomes
\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial (\bar{p}^*/\rho)}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \tau_{ij} \right],
\]
(11)

where \( \tau_{ij} \) is the combination of the viscous and the subgrid-scale stress term, in which the subgrid-scale diffusion coefficient is expressed in the form suggested by Smagorinsky (1963) as
\[
v_i = \left( C_d \Delta \right)^2 \frac{\bar{S}_{ii}}{2},
\]
(12)

where \( \Delta \) is the characteristic length of the computational cell and \( C_d \) is the Smagorinsky constant (0.1).

Equations (5) and (11) can be presented in a conservative form as
\[
\frac{\partial G}{\partial t} + \nabla \cdot \bar{F} = 0.
\]
(13)

The finite-volume computation can proceed using an integration over a specific control volume \( (u) \) as
\[
\int_{S_u} \frac{\partial G_m}{\partial t} dV + \int_{V_u} \nabla \cdot \bar{F} dV = 0.
\]
(14)

By the divergence theorem, one has
\[
\frac{\partial G_m}{\partial t} + \frac{1}{\nu} \int_{S_u} \bar{n} \cdot \bar{F} dS = 0.
\]
(15)

where \( G_m \) is the mean quantity referred to the center of the volume (computational cell) and \( \bar{n} \) is the normal surface vector. By knowing the \( G_m \) quantities of the flow field at a starting time step, the integral form of Equation (15) can be used to calculate the change of \( G_m \) within an elapsed period to further update the \( G_m \) values for the next time step based on an explicit predictor-corrector scheme (MacCormack 1969). During the computation process, the time increment is limited by the CFL criterion (Courant, Friedrichs, and Lewy 1967). Typically, the dimensionless time increment and the period for the unsteady flow computation are, respectively, \( 2.7 \times 10^{-3} \) and 800.

### 4.2. Computational domain and meshes

In all the cases of flow simulations, 3-D computations were performed in the study. The vertical size of the computational domain was selected as 10\( H \) (\( H \) is the slope height). Upstream and downstream of the slope surface, 8\( H \) and 25\( H \) were taken, respectively, in the longitudinal (\( x \)) direction. In the transverse (\( y \)) direction, the size of the domain is 4\( L_a \). Figure 3 shows an example of the computation domain and the corresponding mesh system (608 \( \times 200 \times 71 \)) typically in the case of a ridge. The minimum size of the computational mesh is 0.05\( H \). Considering both the efficiency and result accuracy of the 3-D computations, the

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**Figure 3.** Typical computation domain and mesh system in the case of a ridge.
The size of the tunnel test section is 2.6 m in height and 4.0 m in width. The maximum wind speed in the test section is 35 m/s.

Three models subject to an escarpment, a ridge, and a hill with a slope ratio \( \frac{H}{L} \) of 0.5 were set in the test section of the tunnel. Figure 5 shows the model setup. The slope surface was precisely made by ABS plastic and the height of the slope is 0.15 m. The boundary layer thickness and edge velocity of the approaching flow are, respectively, 1.20 m and 14.0 m/s. A constant temperature anemometer (Dantec StreamLine 90n10) with a cross-film probe (55R61) was used to measure the velocity profiles at various sections. The sampling rate is 1000 Hz and the measurement period is 65 s.

### 4.3. Boundary specifications

In the computations, appropriate pressure and velocity values were specified at exterior phantom cells outside the boundaries to reflect the correct physical nature of the boundaries. At the ground, a no-slip condition was used. At the top, side, and downstream-end boundaries, the values at the phantom cells were specified according to a zero-gradient assumption in the direction normal to the boundary.

At the upstream section, on the other hand, a prescribed power-law velocity profile with a zero-gradient pressure condition was imposed. At each step of computations, the instantaneous upstream velocity profiles were generated numerically by the MDSRFG (modified discretized and synthesizing random flow generation) method (Castro, Paz, and Sonzogni 2011). The anisotropic velocity fluctuations in the three spatial directions were synthesized by the following superposition of harmonic functions,

\[
 u(x, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left[ a_{m,n}^0 \cos \left( k_{m,n}^0 x + \omega_{m,n}^0 \frac{t}{\tau_0} \right) + b_{m,n}^0 \sin \left( k_{m,n}^0 x + \omega_{m,n}^0 \frac{t}{\tau_0} \right) \right],
\]

with \( \omega_{m,n} \in N(0, 2\pi f_m) \), \( \tilde{x} = x/L_z \) and \( L_z = \theta_1 \sqrt{L_x^2 + L_y^2 + L_w^2} \) is the scaling factor for spatial correlation. \( \tau_0 = \theta_1 L_z/U \) is a parameter introduced to allow to control the time correlation. The wave number vector \( k_{m,n}^0 = k_{m,n}/k_0 \) is the three-dimensional distribution on the sphere of inhomogeneous and anisotropic turbulence. For more details of the inlet flow generation, one can refer to the work by Li et al. (2015). Figure 4 illustrates the mean and fluctuating velocity profiles at the inlet cross section. By regression, the power-law \( \alpha \) value is equal to 0.15.

### 5. Wind tunnel experiments

Model experiments were conducted at the ABRI (Architecture and Building Research Institute) wind tunnel and the measurement results of the mean velocity profiles were used to confirm the accuracy of the numerical predictions. The size of the tunnel test section is 2.6 m in height and 4.0 m in width. The maximum wind speed in the test section is 35 m/s.

Three models subject to an escarpment, a ridge, and a hill with a slope ratio \( \frac{H}{L} \) of 0.5 were set in the test section of the tunnel. Figure 5 shows the model setup. The slope surface was precisely made by ABS plastic and the height of the slope is 0.15 m. The boundary layer thickness and edge velocity of the approaching flow are, respectively, 1.20 m and 14.0 m/s. A constant temperature anemometer (Dantec StreamLine 90N10) with a cross-film probe (55R61) was used to measure the velocity profiles at various sections. The sampling rate is 1000 Hz and the measurement period is 65 s.

### 6. Verification

Figures 6 and 7 show the comparisons of the experimental and numerical results in terms of the longitudinal mean and maximum blockage ratio (in the escarpment and ridge cases) is 10%. Preliminary test results have revealed that the relative error lead by this selection is no more than 3%.
Lh = 0.1–0.6) with an atmospheric boundary-layer approaching flow subject to ground condition of Exposure C with a boundary layer thickness of 300 m and a power-law $\alpha$ value of 0.15. The corresponding Reynolds number ($Re$) is $10^7$.

7. Results

Additional series of flow simulations were conducted in the cases of escarpments, ridges, and hills at six slope ratios ($H/L_h = 0.1–0.6$) with an atmospheric boundary-layer approaching flow subject to ground condition of Exposure C with a boundary layer thickness of 300 m and a power-law $\alpha$ value of 0.15. The corresponding Reynolds number ($Re$) is $10^7$.

7.1. Longitudinal mean velocity profiles at various cross-sections

Based on the numerical results, Figures 8–10 illustrate the predicted mean velocity profiles at different slope ratios in the cases.
of an escarpment, a ridge, and a hill, respectively. By excluding the effect of wind directionality and taking the importance factor ($I$) as unity for simplicity, the corresponding velocity profiles suggested by the code are also shown for comparisons.

In the cases of the escarpment (Figure 8), it was found that near the section where the slope begins ($x/L_h = -2$), some velocity deficits are found in the predicted profiles. This is considered to be due to the effect of local longitudinal adverse pressure gradient, which can be commonly found in contracting flows (Fang 1997). A similar phenomenon is also detected in the ridge cases (Figure 9). However, this tendency appears insignificant in the 3-D hill cases (Figure 10).

The predicted results in Figures 9 and 10 show that at the leeward of the ridge or hill, the mean flow forms a reverse-flow

![Figure 9. Comparison of calculated and code-suggested mean velocity profiles (ridge).](#)

![Figure 10. Comparison of calculated and code-suggested mean velocity profiles at center plane (hill).](#)

![Figure 11. Typical streamline patterns of the mean flows in a ridge and a hill case ($H/L_h = 0.5$). (a) Ridge; (b) Hill (center-plane).](#)
7.2. Longitudinal root-mean-square velocity profiles at various cross sections

Figures 13–15 depict the predicted root-mean-square velocity profiles of the three types of slopes. In all the cases, the downstream peak value of the fluctuating velocity profile generally increases as the slope ratio \( H/L_0 \) increases. In the escarpment case, the longitudinal variation of the peak fluctuating velocity appears milder (Figure 13) compared with those in the other two slope cases. In the ridge and hill cases (Figures 14 and 15), on the other hand, the magnitudes of the near-ground root-mean-square velocity increase significantly, downstream of the slope apexes. Additional examinations indicate that the promotion of the extent of flow unsteadiness is due to the occurrence of flow separation.

7.3. Features of velocity fluctuations in the re-circulating zone

Except for the escarpment cases, it is found that the flow after the apex of a ridge or a hill behaves unsteadily due to the region at a larger slope ratio. Typically, the streamline patterns in ridge and hill (Figure 11) cases show that reverse flows occur downstream of the slope apexes. Figure 12 also shows the comparison of the calculated mean velocity profiles between the ridge and hill cases. Within the re-circulating zone, in contrast, the code-suggested profiles do not appear to account for the existence of the separation bubble but are relatively fuller.

![Figure 12. Comparison of calculated mean velocity profiles between the ridge and hill (center plane) cases.](image)

![Figure 13. Calculated root-mean-square velocity profiles in the escarpment cases.](image)
occurrence of separation. Based on the time series numerical results, the probability density functions were obtained at selected locations in the re-circulating zone of a ridge and a hill. Typically as $H/L_n = 0.3$, the results (Figures 16) indicate that the velocity fluctuation varies in a manner close to a normal distribution.

### 7.4. Assessment of velocity profiles

As velocity pressure is directly related to the corresponding wind velocity, the fullness of the velocity profile is important in affecting the magnitude of the resulting wind loads in the along-wind direction. To assess the applicability of the code, comparisons were made in terms of a displacement thickness, defined as

$$
\delta_1 = \int_0^H \left(1 - \frac{\bar{u}}{\bar{u}_0}\right) \, dz.
$$

It is noted that a smaller value of $\delta_1$ reveals that the velocity profile is fuller.

Due to the fact that the pattern of the velocity fluctuation is close to a normal distribution (Figures 16), the $\delta_1$ values were then calculated, respectively, according to the code-suggested profile, the predicted mean velocity profile ($\bar{u}$) and the profile

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**Figure 14.** Calculated root-mean-square velocity profiles in the ridge cases.

**Figure 15.** Calculated center-plane root-mean-square velocity profiles in the hill cases.
the range of interest $(-2 < x/L_h < 2)$. In cases with a larger slope ratio (0.2 and 0.6), non-conservative results were found within the range roughly from about $x = -0.7L_h$ to $0.7L_h$ compared to the $\bar{u}$ curve and from about $x = -1.4L_h$ to $2L_h$ compared to the $\bar{u} + 2u'$ curve.

In contrast to the 2-D ridge cases, the conservative extent of the code suggestions appears worse in the 3-D hill cases near the apex location ($x = 0$). Figure 19 indicates that the code suggestions are non-conservative within a major portion of the range of consideration as $H/L_h = 0.1$. As the slope ratio is greater, non-conservative results were found within the range roughly from about $x = -L_h$ to $0.7L_h$ compared to the $\bar{u}$ curve and from about $x = -2L_h$ to $0.9L_h$ compared to the $\bar{u} + 2u'$ curve.

Figure 16. Power density function of the velocity fluctuations at selected locations in the re-circulating zone. ($H/L_h = 0.3$) (a) Ridge; (b) Hill (center-plane).

Figure 17. Comparison of $\delta_1$ variations in the escarpment case.

Figure 18 demonstrates similar comparisons in the ridge cases. When $H/L_h = 0.1$, the code suggestions appear conservative within the range of interest $(-2 < x/L_h < 2)$. In cases with a larger slope ratio (0.2 and 0.6), non-conservative results were found within the range roughly from about $x = -0.7L_h$ to $0.7L_h$ compared to the $\bar{u}$ curve and from about $x = -1.4L_h$ to $2L_h$ compared to the $\bar{u} + 2u'$ curve.
suggestions in the evaluation of the velocity pressure profiles. Under the prescribed conditions of the study, the conclusions are summarized as follows:

(1) Flow separation occurs in the case of a ridge or a hill and results in flow unsteadiness in the leeward region.

8. Conclusions

Turbulent flow fields in areas with isolated ridges, escarpments and hills were analyzed numerically in areas with isolated ridges, escarpments, and hills, respectively. The predicted velocity profiles were used to assess the applicability of the related code

Figure 18. Comparison of \( \delta \) variations in the ridge case.

Figure 19. Comparison of \( \delta \) variations in the hill case.
Within the re-circulating zone, the velocity fluctuation varies generally in a manner close to a normal distribution. In addition, the code-suggested profiles do not appear to reflect the existence of the separation bubble but are relatively fuller than the predicted ones.

(2) To evaluate velocity pressure profiles according to the quasi-steady theory, besides the mean flow, the contribution from the velocity fluctuations can be included to obtain more conservative results for wind load evaluations.

(3) Based on the results of comparisons, it was found that the code suggestions may not be conservative and may require examination for further possible modifications.

**Nomenclature**

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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$c$</td>
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<tr>
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<td>$u'$</td>
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<td>displacement thickness</td>
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**References**


